INCENTIVE DESIGN IN THREE LEVEL HIERARCHIES UNDER MORAL HAZARD

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ABSTRACT

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This thesis studies the incentives in multi-level hierarchical institutions under moral hazard. The principal's objective is to induce the agent exert "high" effort and a supervisor is used to monitor either the agent's effort or the output level. We extend a canonical agent-supervisor-principal model by introducing ex-ante collusion possibilities, whereby the parties can side-contract before execution of the official contract, that is, before the supervisor and the agent incur their respective inspection and effort costs. The thesis characterizes least-cost incentive contracts with and without ex-ante and ex-post collusion possibilities. It is shown that preventing only ex-ante, or only ex-post, collusion does not prevent the other automatically: the two collusion-proofness constraints are independent. Second, when full collusion possibilities are incorporated, the only constraint that can be ignored is the supervisor's incentive compatibility constraint (implied by ex-ante collusion prevention). Third, it is shown that safeguarding against ex-ante collusion raises the principal's expected costs, in some cases "significantly". We discuss the effectiveness of preventing all types of collusion and show that despite of increases in expected costs, the principal still finds preventing all types of collusions optimal. Finally, we show that input monitoring is structurally more efficient than output monitoring. If the same given monitoring technology is available and equally effective in generating hard evidence, the supervisor should assess the effort level of the agent and not the final output.

Keywords: hierarchy, corruption, collusion, incentives, contracts.

ÖZET

ÜÇ KATMANLI HİYERARŞİLERDE AHLAKİ TEHLİKE ALTINDA TEŞVİK TASARIMI

ATAKAN AÇIKGÖZ

Ekonomi, Yüksek Lisans Tezi, Haziran 2018

Tez Danışmanı: Prof. Dr. Mehmet Baç

Bu tez, çok katmanlı (Asil-Denetçi-Vekil) hiyerarşik bir kurumda, Vekil'den arzu edilen seviyede efor elde edilmesini sağlayacak optimal teşvik sistemlerini (ücret/bonus/ceza) incelemektedir. Vekil'in "yüksek" eforda çalışmasını sağlamak için Asil, bir denetçi kullanarak Vekil'in eforunu veya üretim çıktısını ölçebilmektedir. Tez, bu standart modeli, biri efor öncesi ve diğeri efor sonrası olmak üzere iki zararlı işbirliği imkanı ekleyerek zenginleştirmektedir. Efor öncesi zararlı işbirliği, taraflar iş akdi gereği yükümlüklerini yerine getirmeye başlamadan önce, yani, Vekil efor seviyesini seçmeden ve Denetçi gözlem yapmaya başlamadan önce, bu iki tarafın kendi çıkarları gereği varabilecekleri bir anlaşmadır. Efor sonrası zararlı işbirliği ise denetim sonucu ortaya çıktıktan sonra oluşabilmektedir. Bu zararlı işbirliği imkanlarını ortadan kaldırmak, optimal teşvik çözümleri için birer kısıt teşkil etmektedir. Tez'in bulguları şöyle özetlenebilir: 1. Salt efor öncesi veya salt efor sonrası işbirliğini engellemek diğer işbirliği imkanını ortadan kaldırmamaktadır (bu iki işbirliği kısıtı birbirinden bağımsızdır.) 2. Her tür zararlı işbirliği tam olarak engellendiğinde, göz ardı edilebilecek tek kısıt Denetçinin kişisel teşvik kısıtıdır. 3. Efor öncesi işbirliğini önlemek maliyeti (bazı durumlarda belirgin ölçüde) arttırmaktadır. Dolayısıyla işbirliklerinin önlenmesinin etkin olup olmadığını da sorgulayıp, amaç her koşulda Vekil'in efor sarfetmesini sağlamak ise, maliyet artışlarına rağmen bu işbirliklerini önlemenin Asil açısından optimal olduğu gösterilmektedir. 4. Efor gözlemlenmesinin çıktı gözlemlenmesine göre yapısal olarak maliyet açısından daha verimli olduğu gösterilmektedir.

Anahtar Kelimeler: hiyerarşi, yolsuzluk, zararlı işbirliği, teşvikler, sözleşmeler.

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Contents

List of Figures

1 Introduction

The study of incentive design is an important field of research to improve our understanding of the operational problems that are typical to any hierarchical organization. Members should be induced to perform the tasks assigned, but unobservability of actions, known in the literature as "moral hazard", can create significant obstacles to this end. Using supervisors to cope with this problem brings in a new question, concerning the incentives of the supervisor whose actions may also not be observable. Moreover, members of an organization can side-contract to improve their own benefits at the expense of members excluded from the group. This phenomenon, known in the literature as "collusion", leads to the collapse of the incentive structure.

In the economics literature, hierarchy is studied in its simplest three-layer form, the agent-supervisor-principal model. Before moving on to the optimal incentive design in hierarchies, it is useful to overview the agency problem between the agent and the principal. The principal hires an agent to realize a task on behalf of himself because the task may be too complicated or too costly for the principal. In this case, there can exist two main problems due to information asymmetry. First, before the contracts are executed, the principal may not be able to know the agent's ability, effort cost or any other characteristics that are known to agent. This is called the *adverse selection* problem. Better searching mechanisms and contract design can help to deal with this problem. The second problem is *moral hazard* mentioned above, also known as "hidden action". If the objectives of the principal and the agent are in conflict as they usually are, the agent would not behave according to the principal's interest because the agent would pursue his own objective, which is to get the highest wage by exerting smallest amount of effort. It would be naive to expect every agent have top ethical standards. Therefore the agent should be offered wages and rewards that will make it in his own interest to exert the effort the principal expects. The thesis adopts a hidden action set-up where the agent's effort level is only known to himself but the effort cost of the agent is public information.

One solution to the agent's moral hazard problem is monitoring, to collect information about the agent's actions. This monitoring task can be executed either by the principal himself or by a delegated supervisor. The standard practice is to hire a supervisor and delegate this task. However, inclusion of the supervisor into the system creates further problems. Now that we have a hierarchy consisting of three layers, the principal should provide both the agent and the supervisor the correct incentives to perform. But employment of a supervisor creates another problem, the possibility of collusion. For example, the supervisor can accept a bribe from the agent and misreport. This is one type of collusive behaviours and it can take many other forms in the organisation. Public officials accepting bribes (colluding with clients) to give unjust permits can illustrate a collusion in the government hierarchies. If the incentive mechanisms are not properly designed, corruption can happen and, sometimes, produce disastrous consequences for the organization.[1]

The main contribution of this thesis lies in introducing new sets of collusion constraints to evaluate their impacts on the design of incentives in hierarchies, under moral hazard. Collusion can be defined as a bilateral, hidden arrangement involving transfers, whereby a coalition in the hierarchy (group of members) forms an agreement to undertake specific actions so as to raise its members' joint and individual benefits. In the typical three-layer hierarchy model, these coalitions consist of two parties.

As all agreements, collusion must be enforceable. All static models assume that collusion can be enforced, some with a cost, some without a cost. But the mechanism through which the parties can enforce collusion, how the parties prevent each other's deviation from the side contract, is left unmodelled. In this thesis we adopt the same approach. In our static model, side-contracting (collusion) occurs whenever the parties' total utility is larger than without side-contracting.[2]

The new set of collusion constraints introduced in this thesis is "**ex-ante**" in the sense that the opportunity to collude arises before the parties engage in their assigned tasks, as opposed to the standard (**ex-post**) opportunity to collude after the tasks are complete but before the contracts are executed (such as, suppression of information in a report used in determination of wages to be paid). The questions we study and a summary of our results are in order below.

First, since we have now two different types of collusion, relationship between them has to be examined. We ask whether preventing one type of collusion, automatically prevents the other one. Characterization of full collusion proof contracts show that none of the collusion constraints can be ignored, depending on the parameter values, either one of the four collusion constraints (ex-ante downward, ex-ante upward, ex-post downward, ex-post upward) can be binding.

[1]Although not synonymous, corruption is a form of collusion. We care about corruption since it leads to eradication of confidence in the society. If an officer in the judicial system gets involved in corruption, it may ultimately result in collapse of the system. Also, corruption reduces the reliability and prestige of the country. As a direct result of this, foreign capital flow into the country may decrease which can put pressure on economy.

[2]In real-world cases, various mechanisms are available to enforce collusion, essentially based on repetitive encounters between the colluding parties such as reciprocity and and face-to-face relations. In these environments, mutual credible threats for deviations, if available, can serve to enforce collusion.

Secondly, having included the ex-ante collusion proofness constraints into the principal's problem, we observe that the supervisor's incentive compatibility constraint is automatically satisfied. This does not hold if one ignores the ex-ante collusions are ignored. Specifically, preventing ex-ante downward collusion as a by-product ensures that the supervisor has the incentive to monitor the agent. In other words, the principal does not have to worry about getting the supervisor monitor the agent if the contracts are full-collusion-proof.

We also ask if preventing ex-ante collusion possibilities have any effect on the principal's expected costs. To find the answer, we solve the principal's problem without and with ex-ante collusion constraints. Our results show that, there is a raise in expected costs which for some specific parameters, is doubled. Obviously this is not good news for the principal; now, higher wages must be paid to prevent all types of collusion. This leads us to another question, as to whether the rise in the expected costs to prevent all collusion is financially justifiable, an issue which we tackle next.

We observe that in this hierarchical environment, the principal has four possible strategies in designing contracts: preventing all types of collusion, permitting ex-post downward collusion, permitting ex-post upward collusion and permitting both ex-post collusions. Ex-ante collusions must be prevented at any cost because the principal's objective is to induce high effort (as if high output is infinitely valued). We solve for optimal contracts under each remaining strategy and show that despite of raise in expected costs, preventing all types of collusion is the weakly optimal strategy for the principal.

Lastly, we define an alternative monitoring system, input monitoring, where the supervisor monitors the effort level of the agent instead of the output the agent produces. The monitoring technology (specifically, the probability of obtaining hard evidence and the cost of monitoring) is identical. We show that input monitoring is structurally more efficient than output monitoring. The level of output is stochastically related to effort level. Because the objective is to induce high effort, monitoring effort is more effective than a variable, like output, that is correlated with it. In more detail, for input monitoring, all the wages can be reduced if monitoring cost gets smaller. However, inefficiency of the effort in output monitoring prevents the principal reduce all the wages proportionally to monitoring cost. If monitoring cost is sufficiently small, ex-post upward collusion constraint is binding (not the case in input monitoring) so that wages have to be kept above a certain level.

2 Literature Review

The related literature, in the broadest sense, includes models of moral hazard and the theory of incentives. Organisations are made of nested hierarchies which represent ranking of authorities or flow of information−these topics, though important, are outside the scope of this thesis. Analyses of incentive mechanisms in vertical hierarchies in the economics literature are first done in principal-agent models. Those with hidden action deal with imperfect information after the contracts are written. The agent may choose an unobservable effort level or the agent learns his effort cost.

Without moral hazard, i.e., when effort is observable and contractible, the design of optimal contracts is fairly simple since the principal can directly induce the desired effort level of the agent. Under moral hazard, however, the agent's action is not observable to the principal. This raises a problem of designing contracts that offer effort incentives by relating wage payments to some observable variable that is correlated with the agent's effort. When feasible, such contract design will raise the expected wage bill. Ultimately, under moral hazard, to compensate for costly effort, *incentive constraint* has to be satisfied and to induce voluntary participation, *participation constraint* has to be satisfied. The contracts that satisfy these two conditions are called incentive feasible contracts. In this context, these types of contracts are valuable for us since optimal contract that minimized the cost of implementation should be among these contracts. In the absence of the supervisor, assuming that the output is observable, the contracts should be contingent on the level of production. Agent's wages should increase as the profit of the principal increases or the output level increases, its functional form can be linear or non-linear depending on the model.

The idea to include into the model a supervisor to acquire information about agent's action as a potential solution for hidden action is formally studies by, first, Tirole (1986). Tirole models the supervisor as an intermediary player and points out that inclusion of this third party leads to collusion possibilities in the hierarchy. The supervisor has her own interests, just like the agent, which opens the door for information manipulation. Since the principal relies on the information that the supervisor acquires, the agent may simply offer a bribe to the supervisor to reveal favourable information and conceal unfavourable information. Introduction of the supervisor has thus created a new collusion and corruption literature, which has expanded since then.

However, because it brings in multiple collusion possibilities that are costly to prevent, the inclusion of the supervisor into the hierarchy needs to be justified, financially. Initially, the supervisor was justified by the assumption that either the principal has no time to conduct the supervision or the supervisor is much more efficient in monitoring. However, its possible economic benefit was analysed after a time. Regarding this issue, Tirole (1992) compares two and three level hierarchies and conclude standard sufficient statistics principles for rewarding agents do not hold in the presence of collusion. Thus, threat of collusion may get ahead of benefit of supervisor. Baliga (1999) proposes a new method to make use of the supervisor. If the supervisor and the agent himself gives a report about the type of the agent to the principal at the same; and the agent paid is lower if the reports do not match, comparison of the optimal contracts justifies the economic benefit of the supervisor.

Strausz (1997) addresses the question of delegation of monitoring directly, comparing two strategies for the principal within a canonical hierarchy set-up which includes hidden action. The first strategy is that the principal hires a supervisor to monitor the agent, the second is that he conducts the monitoring himself. The monitoring technology is the same under both strategies and the analysis focuses on the costs of incentives provided to agent. He proves that hiring a supervisor and incentivizing the agent through adjustment of two contracts for two people is easier than doing with one contract for just the agent. If the principal chooses to monitor himself, the agent would infer that the principal would not ever reveal a high output evidence. Under the alternative arrangement, the principal gives incentives to the supervisor to reveal high output, which relaxes the agent's incentive constraint.

Recent research in this field utilizes three-level incentive schemes, which create collusion possibilities as we mentioned earlier. In the remainder of this section we focus on the collusion literature.

There is a considerable literature trying to understand the effects of supervision, methods to minimize the wage bill while inducing all the desirable actions by the agents. As mentioned, the new aspect in this thesis is introduction of collusion possibilities before the members of an organization undertake their respective tasks. For instance, a police officer and his supervisor can agree to collude ex-ante, share the corrupt proceeds from the Mafia and in return of a bribe, ignore the activities of the Mafia. To our knowledge the possibility of ex-ante collusion has by and large been ignored.

Whereas the main focus in the literature is on potential side transfers between the agent and the supervisor, collusion possibilities between the supervisor and the principal are also recognized. The only type of collusion admitted in extant models is (what we call) ex-post collusion. This type of collusion occurs when the supervisor acquires an information about the agent's performance and agrees, with one of the other players, to reveal a different finding. For instance, if the supervisor has evidence justifying the low output, the agent has the possibility of approaching to the supervisor, offering a bribe so as the latter does not reveal the evidence. If the offer is accepted, side transfers happen and we have collusion. In the absence of these collusion possibilities the principal's expected wage bill would be much lower. When collusion possibilities are included, however, contracts must be adjusted accordingly. Tirole (1986) establishes the basis of the collusion argument by combining sociology and economics. Collusive behaviour in the sociology literature has deep roots but economic analysis is recent. His economic analysis establishes that the possibility of collusive behaviour decreases the efficiency of hierarchical systems, but as a threat it should be banished by an appropriate incentive mechanism. He warns, however, that this conclusion is an extreme one and that it should be assessed cautiously, for there can be a case for beneficial collusion, where side transfers are required to maintain long-term relationships in any level of the hierarchy.

Kofman and Lawarrée (1993) study a potential solution to collusion between the agent and the supervisor. They introduce an external supervisor (called, auditor) whose main aim is to prevent deviation of the internal supervisor. The external supervisor has short term contracts, so she can bear much easier than the first one to the pressures from the organisation so that they assume external supervisor never colludes. This makes the external supervisor more reliable but she may not be able to know specific requirements of the job as well. Thus, the benefits from using an external supervisor are ambiguous. They show that optimal contracts may indeed require randomly assigned external auditors.

In another paper, Kofman and Lawarrée (1996) argue the potential benefits of allowing collusion by using a similar canonical model. They assume the auditor can have two types: dishonest and honest. This information is unknown to the principal, so decision of allowing or deterring collusion should be made under information asymmetry. They show that preventing collusion may not be efficient since both dishonest and honest auditors are rewarded. Also, allowing collusion is costly since dishonest auditors take advantage of the situation and deters. Thus, depending on the characteristics of the auditor optimal contracts may change. They show that, if there is a positive probability of hiring a dishonest auditor, there may be some specific instances where permitting collusion turns out to be optimal. Moreover, if there are high punishments for low output, permitting collusion is always the most efficient choice for the principal. Note that, they also show hiring an auditor when collusion is allowed still efficient since the manager has to pay bribe to make the auditor reveal a favourable report. There exist other papers offering solutions to ex-post collusion, analysing the effect of collusion possibilities on efficiency or beneficial collusions. However, all these papers are limited to the case of ex-post collusion.

Ex-ante collusion possibilities are introduced in hierarchy models of moral hazard by Bac (1996) and later by Bac and Kucuksenel (2005). This kind of collusion occurs when the supervisor stops monitoring the agent in return of a bribe. In a way, the supervisor is taken out of the model and there is no chance to produce a report on the agent's performance. Since this collusion occurs before not their tasks are done, it is called ex-ante. Whether there is an actual threat of ex-ante collusion or not was unknown until recently. Bac and Kucuksenel (2005) extended the Tirole's (1986) paper by introducing ex-ante collusion and tried to examine the interaction (if any) between these new ex-ante type of collusion with ex-post collusion, along with the incentive constraints of the players in the hierarchy. Their analysis proves that if probability of the detection is large and monitoring costs are small, ex-post collusion-proof contracts automatically become ex-ante collusion-proof, so that in those cases ex-ante collusion can be ignored. Otherwise, ex-ante collusions can be prevented by increasing the wages paid when there is productive evidence or by decreasing the wages when there is no evidence. When ex-ante possibilities are taken into account, adjusting wage gaps provides the required incentives to protect the order in the hierarchy. They also note that, if the supervisor stops monitoring, ex-post considerations becomes irrelevant as well since there remains no possibility to deviate to better state for the supervisor ex-post. This is an important implication coming from the interaction of two types of collusions. The thesis incorporates some of the findings of this paper and tries to advance the analysis of ex-ante constraints further.

Though preventing collusion by contract design seems the obvious solution, the costs can be high. Permitting some types of collusions can help the principal to reduce to expected costs and can be chosen provided the agent's incentive to exert high effort is maintained. Vafai (2018) addresses this issue in a standard three-layer hierarchy model, with two ex-post collusion possibilities: ex-post downward collusion and ex-post upward collusion. Ex-post downward collusion occurs when the supervisor finds low output and the agent bribes the supervisor to deviate to an empty report. It is sensible to prevent this collusion because otherwise incentivizing high effort by the agent will be difficult and costly. This is the common type of collusion examined in the literature. The other, upward collusion "happens when the supervisor finds high output and is approached by the principal to deviate to empty report." A priori, this kind of collusion has an ambiguous impact on effort incentives. Vafai identifies four strategies for the principal: permitting both types of ex-post collusions, preventing only ex-post downward collusion, preventing only ex-post upward collusion and preventing both types of ex-post collusions. He then proves that the optimal strategy is to prevent all types of collusions, by comparing the expected costs of these afore mentioned strategies. He argues that permitting upward collusion increases the expected costs through two main channels. First, the principal has to pay more to the agent to guarantee that he exerts high effort (called incentive effect); second, preventing downward corruption becomes harder, because under upward ex-post collusion the agent knows that he will never ever be paid the high wage, that is, he will be aware of the fact that his efforts can at best produce an empty report (called downward corruption effect.)

The basic model in this thesis borrows from Vafai (2018). It extends the collusion possibilities in the hierarchy and studies input and output monitoring cases separately. This extension would not have any impact if supervision were costless. Introduction of a positive monitoring cost for the supervisor, seemingly a minor modification, is shown to have important implications on the optimal contracts, in particular under the possibility of ex-ante collusion. Showing this, the thesis proceeds with a comparison of the principal's utility (expected wage bill) from from four different strategies consisting of permitting and preventing ex-post collusion both exclusively and together. It turns out that preventing all kinds of collusion would be in the best interest of the principal, if high effort is expected from the agent. Thus, our results agree with Vafai's findings, preventing all types of collusion remains as weakly optimal strategy for the principal. In this way, by considering ex-ante collusion possibilities along with ex-post ones, we have strengthened this conclusion.

Another subject of research is the comparative analysis of different types of monitoring. Among the limited number of contributions, to our knowledge, Khalil (1995) is the first to analyse the differences and compare the effectiveness of different monitoring methods. Khalil (1995) uses a principal-agent model where the principal monitors the agent. He argues that residual claimancy is the source of rent in the hierarchy and the choice between input and output monitoring is determined according to the identity of the residual claimant. If the principal is the residual claimant, then input monitoring is efficient, otherwise output monitoring is preferred.

Zhao (2008) uses a model where the agent has multiple task and the supervisor monitors the agent. He shows that multi tasks and limited liability constraints make the output-based incentive system preferable. Rewarding the overall outcome becomes better option than evaluating piecewise effort level of the agent. He argues that these results rationalize output-based performance bonuses. Although models in these last two papers are completely different than ours, they illustrate the large variety of approaches used in the literature. We also carry out a comparison, output vs. input monitoring, in this thesis.

The thesis is organized as follows. In the next section, we present the model which is an extended version of a canonical agent-supervisor-principal model. In section 4, we introduce ex-ante collusions and analyse their effect on optimal contracts under output monitoring. In section 5, we check whether permitting ex-post collusions or preventing any type of collusion is better strategy for the principal. In section 6, we suggest an alternative monitoring method and conduct its analysis. In section 7, we present some of the results that are generally coming out of comparison between these two types of monitoring. Lastly, section 8 concludes the thesis with a summary of results.

3 The Model

The model is an extension of the canonical agent-supervisor-principal setup introduced by Tirole (1986) and studied by many others later on. The hierarchy consists of three members, the agent, the supervisor and the principal. I assume that all parties are risk neutral and that their outside options are (normalized to) zero.

3.1 Tasks, utilities and contracts

The agent's task is to exert effort, but his actions are unobservable. He can exert high effort $e = 1$ which costs him c_e, or exert low effort $e = 0$, at zero cost. Effort produces an output according to a stochastic technology: If $e = 1$, output is high, $x_H > 0$, with probability $\pi \in (0, 1]$ and low, $x_L = 0$, with probability 1-π.

Output, like the agent's effort, is not directly observable. The supervisor' task is to monitor the agent's output or the effort input, (depending on the case or, choice of the principal) and submit a report r on the inspection result to the principal. Monitoring costs c_m to the supervisor and generates verifiable (hard) evidence with probability μ about the target variable, effort or output. With probability $1 - \mu$ monitoring fails, that is, she obtains no evidence.

The supervisor's choice of action is also unobservable, which brings in a second moral hazard issue to solve for the principal. She must find it in her own interest to monitor the agent and report it to the principal. If the supervisor does not monitor the agent, she cannot obtain any evidence about the target variable. Hard evidence cannot be fabricated, but note that it can be concealed.

To illustrate, in the case of output monitoring, if the supervisor chooses to monitor, she generates hard evidence about output, either x_H or x_L, with probability $\mu \in (0, 1)$. Then, the supervisor's report r can be of three types, $r = x_L$, $r = x_H$ and $r = \emptyset$. If she does not monitor output, the only possible report is $r = \emptyset$.

Denoting the agent's wage by w and the supervisor's wage by s, final utilities are $U^A(w, e) = w - c_e$ for the agent if he exerts effort, $U^A(w, 0) = w$ if he does not, and $U^S(s, m) = s - c_m$ for the supervisor if she monitors, $U^S(s, 0) = s$ if she does not. The principal's objective is to induce the agent to exert high effort at minimum expected cost.

The sequence of events is shown in Figure 1 below. The principal offers a pair of contracts $C_A = \{w_L, w_\emptyset, w_H\}$ for the agent and $C_S = \{s_L, s_\emptyset, s_H\}$ for the supervisor, each specifying a wage pair (w_r, s_r) for each possible output report r. Following acceptance of the contracts but before the supervisor and the agent undertake their respective

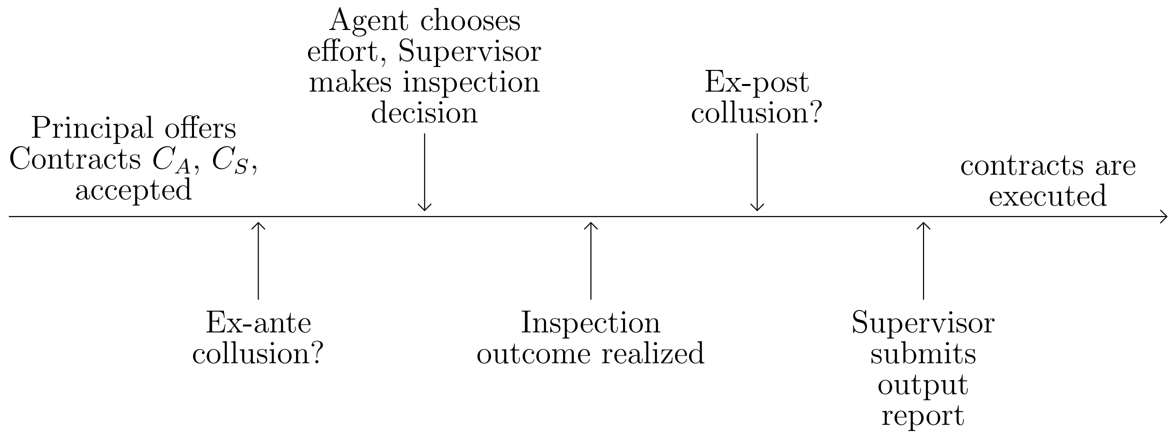

Figure 1: The sequence of events in the hierarchy.

tasks, any pair of the three parties can engage in collusion. This kind of side contracting may occur before effort and monitoring choices, hence the label "ex-ante collusion." In the next phase the agent chooses his effort, following which the supervisor decides on whether to monitor. There is another collusion possibility at this stage, before the supervisor submits her report. Based on the information she obtained, the supervisor can approach the agent or the principal to jointly raise their final utilities by suppressing hard output evidence, if any. This kind of side contracting is called "ex-post collusion." Finally, the supervisor submits a report, on the basis of which contracts are executed.

We assume that the agent and the supervisor are protected by limited liability, that is, their wages in each outcome cannot be reduced below a lower bound, which we take equal to zero.

$$w_L \geq 0,\ w_\emptyset \geq 0,\ w_H \geq 0,\ s_L \geq 0,\ s_\emptyset \geq 0,\ s_H \geq 0. \tag{1}$$

3.2 Collusion possibilities in the hierarchy

Ex-post the supervisor has an informational power (the outcome of output monitoring) which she can abuse in side contracting with either the agent or the principal, depending on the hard information she got. She can offer the agent or the principal to suppress the hard evidence for a transfer, a bribe.

Ex-ante, before even the agent and the supervisor perform their tasks, the motivation for collusion is completely different. There is scope for beneficial agent-supervisor side contracts because the two can jointly deviate to shirking and economize on the costs of their projected actions, effort and monitoring. On the other hand the principal can collude with the supervisor against the agent, whereby the supervisor deviates to shirking for a bribe from the principal and the latter so economizes on the wage bill. The exact forms of these collusive agreements will be explained in the sequel.

For simplicity, the analysis assumes that all types of collusion are costlessly enforced and implemented. Thus, the parties will collude whenever their total expected utilities are larger than without collusion. Obviously this brings and upper bound on the utilities that the parties can reach via collusion and thus a lower bound on the principal's utility from preventing all types of collusion while inducing the agent exert high effort.

4 Supervisor monitors output

The analysis proceeds in two steps. First, as a benchmark we study the optimal contracts without (hence, ignoring) the ex-ante collusion possibilities. The second part will incorporate the ex-ante collusion proofness constraints and highlight their impact on both the optimal contracts and the principal's expected wage bill.

4.1 Optimal ex-post collusion-proof contracts

This subsection states the parties' expected utilities, derives the incentive compatibility constraints and the optimal contracts C_A and C_S that are ex-post collusion-proof.

Assume that the supervisor monitors output. The agent's incentive compatibility constraint when ex-post collusion does not occur is

$$\mu[\pi w_H + (1-\pi)w_L] + (1-\mu)w_\emptyset - c_e \geq \mu w_L + (1-\mu)w_\emptyset.$$

The left hand side is the agent's utility when he exerts effort and the right hand side is the utility from shirking (note that the supervisor may not be able to generate hard evidence about the output, even though output is low). This constraint can be simplified as

$$w_H - w_L \geq \frac{c_e}{\mu\pi}. \tag{2}$$

Thus, to motivate the agent the contract must set at least a difference of $\frac{c_e}{\mu\pi}$ between the agent's wages under high and low output reports.

The supervisor must be induced to monitor the agent, for otherwise the only possible output report is $r = \emptyset$ and hence the agent has no incentive to exert effort. Assume that the agent exerts high effort and the contracts are ex-post collusion-proof, the supervisor's incentive compatibility constraint is

$$\mu[\pi s_H + (1-\pi)s_L] + (1-\mu)s_\emptyset - c_m \geq s_\emptyset.$$

With probability μ monitoring is successful and the supervisor's expected wage is $\pi s_H + (1-\pi)s_L$, while with probability $1-\mu$ monitoring fails and his wage is $s_\emptyset$. Thus the left hand side is the expected utility of the supervisor when she monitors and the right hand side is the utility from not monitoring, which is simply $s_\emptyset$. The supervisor's incentive compatibility constraint simplifies to

$$\mu[\pi s_H + (1-\pi)s_L] \geq \mu s_\emptyset + c_m. \tag{3}$$

It is easy to see that the limited liability constraints in (1) imply that the contract automatically satisfies the participation constraints of the agent and the supervisor.[3]

Consider now the two collusion possibilities, ex-post. First, if the supervisor obtains low output evidence, the agent can offer a bribe to the supervisor so that the latter submits the report $r = \emptyset$ instead of $r = x_L$. Under an empty report, the total utility of the agent the supervisor is $s_\emptyset + w_\emptyset$. Assuming that the supervisor does not participate in collusion when he is indifferent, *ex-post downward collusion* is prevented if

$$s_L + w_L \geq s_\emptyset + w_\emptyset. \tag{4}$$

Second, when the supervisor obtains high output evidence, she may collude with the principal who would offer a bribe to the supervisor to withhold the information and report $r = \emptyset$ instead of $r = x_H$. Because the supervisor's wages are direct costs for the principal, the surplus from this type of collusion depends solely on the agent's wages.[4] *Ex-post upward collusion* cannot occur if the agent's wage under $r = \emptyset$ is at least as large as his wage under $r = x_H$:

$$w_\emptyset \geq w_H. \tag{5}$$

When (4) and (5) hold so that the hierarchy is protected against downward ex-post and upward ex-post collusion, the principal's expected wage cost EC can be written as

$$\begin{array}{ll} \min\limits_{w_L, w_\emptyset, w_H, s_L, s_\emptyset, s_H} & \mu[\pi(w_H + s_H) + (1-\pi)(w_L + s_L)] + (1-\mu)(w_\emptyset + s_\emptyset) \\ \text{subject to} & (1), (2), (3), (4) \textit{ and} (5) \end{array}$$

The solution to this problem is stated and explained below.

Proposition 1 *Suppose that ex-ante collusions are not possible. The optimal ex-post collusion-proof contract and the principal's corresponding expected cost of inducing high effort are:*
(i) $(w_L^X, w_\emptyset^X, w_H^X) = (0, \frac{c_e}{\mu\pi}, \frac{c_e}{\mu\pi})$, $(s_L^X, s_\emptyset^X, s_H^X) = (\frac{c_e}{\mu\pi}, 0, 0)$ *and* $EC^X = \frac{c_e}{\mu\pi}$ *if* $\frac{(1-\pi)}{\pi} c_e \geq c_m$*;*
(ii) $(w_L^X, w_\emptyset^X, w_H^X) = (0, \frac{c_e}{\mu\pi}, \frac{c_e}{\mu\pi})$, $(s_L^X, s_\emptyset^X, s_H^X) = (\sigma_L, 0, \sigma_H)$ *such that* $\mu[\pi\sigma_H + (1-\pi)\sigma_L] = c_m$ *satisfying* $\sigma_L \geq \frac{c_e}{\mu\pi}$ *and* $EC^X = c_m + c_e\frac{(1-\mu+\mu\pi)}{\mu\pi}$, *if* $\frac{(1-\pi)}{\pi} c_e \leq c_m$.

Thus, the agent is paid a bonus to cover his effort cost in the two possible outputs

[3] The participation constraints are $\mu[\pi w_H + (1-\pi)w_L] + (1-\mu)w_\emptyset - c_e \geq 0$ for the agent, $\mu[\pi s_H + (1-\pi)s_L] + (1-\mu)s_\emptyset - c_m \geq 0$ for the supervisor.

[4] Stated differently, the principal can at most offer the supervisor the bribe $b = w_H - w_\emptyset + s_H - s_\emptyset$ for reporting $r = \emptyset$ instead of $r = x_H$, which the supervisor would accept if b is larger than the wage s_H she gets by reporting $r = x_H$. Thus collusion will not happen if $s_H \geq s_\emptyset + (w_H - w_\emptyset + s_H - s_\emptyset)$ which yields the collusion-proofness constraint above.

x_H and $\emptyset$ under high effort. The zero wage paid under hard evidence of low output keeps the agent on the high effort track, at minimum cost. As for the supervisor's optimal contract, Proposition 1 distinguishes between two cases. If the monitoring cost of the supervisor is below a threshold $\frac{(1-\pi)}{\pi}c_e$, we are in case (i): Supervisor's incentive compatibility constraint becomes redundant, hence she is only paid when the output is low and that is the minimum amount that satisfies (4).

If $c_m \geq \frac{(1-\pi)}{\pi}c_e$ that is case (ii): The principal has no choice but to increase low or high output wages of supervisor to satisfy her incentive compatibility constraint and ensure that she monitors the agent. Otherwise, the supervisor will deviate and stop monitoring. Thus, s_L and s_H must each be non-negative and satisfy $\mu[\pi s_H + (1-\pi)s_L] = c_m$. Increasing these wages further is not optimal, so that we set this specific combination of s_L and s_H equal to c_m. Also, s_L has to be at least $\frac{c_e}{\mu\pi}$ to satisfy (4). Below this level, downward corruption occurs. Any combination of these wages satisfying these two specifications will be optimal for the principal.[5] Another thing that should be noted that, this threshold depends on effort cost of agent and π value. If c_e is higher or π is smaller then, it is more likely that c_m will not bind since increase in those constraints lead to higher s_L wage.

4.2 Optimal full collusion-proof contracts

Both exerting effort and monitoring are costly activities. Thus, their utilities decrease if they complete their tasks. The agent can approach to the supervisor and propose not to realize their tasks jointly in return of a bribe. Note that, if there is no extra bribe, supervisor already may choose not to monitor. This type of side contracting is called ex-ante downward collusion. There is also ex-ante upward collusion possibility between the principal and the supervisor. The principal bribes her with the surplus that will come from agent's expected and realized wage due to worse report. In the following subsections, we introduce these two new constraints and show their effects on the optimal contracts if there are any.

4.2.1 Ex-ante downward and ex-ante upward protected contracts

We begin by generating the ex-ante constraints. Agent and supervisor can make agreement before supervisor monitors and agent puts effort meaning that they can simultaneously set e=0 and m=0. This type of collusion is called ex-ante downward collusion.

[5] More precisely, when s_L is minimal and equal to $\frac{c_e}{\mu\pi}$ the principal sets $s_H = \frac{c_m}{\mu\pi} - \frac{(1-\pi)}{\pi}\frac{c_e}{\mu\pi}$, whereas if $s_H = 0$ then s_L is maximal and equal to $\frac{c_m}{\mu(1-\pi)}$.

In this case, if there is surplus compared to the their normal expected utilities, corruption occurs. Therefore, total expected utilities of the agent and the supervisor from trustworthy reposting, $\mu[\pi s_H + (1-\pi)s_L] + (1-\mu)s_\emptyset - c_m + \mu[\pi w_H + (1-\pi)w_L] + (1-\mu)w_\emptyset - c_e$, should exceed the total utility of the agent and the supervisor when they engage in corruption that is $s_\emptyset + w_\emptyset$. Then, the institution will not be vulnerable to ex-ante downward collusion. altogether, ex-ante downward collusion constraint is $\mu[\pi s_H + (1-\pi)s_L] + (1-\mu)s_\emptyset - c_m + \mu[\pi w_H + (1-\pi)w_L] + (1-\mu)w_\emptyset - c_e \geq s_\emptyset + w_\emptyset$ which is simplified as

$$\mu[\pi s_H + (1-\pi)s_L] + \mu[\pi w_H + (1-\pi)w_L] \geq \mu s_\emptyset + \mu w_\emptyset + c_m + c_e \tag{6}$$

By including this constraint principal make sure that agent and supervisor at least will not engage in corruption before they do their duties. Next, we check whether this constraint has an effect on wages for both parties. To achieve this principal solves the minimization problem in 4.1 with additional constraint (6). As we ignored both ex-ante constraints for ex-post collusion-proof contracts, we do not consider the ex-ante upward collusion for now. It helps us to see the isolated effect of ex-ante downward constraint and also if we permit ex-ante upward collusion, the agent does not exert effort which is not desired.

Moreover, after principal offers contracts, he can directly try to bribe the supervisor for not monitoring the agent which results in ex-ante upward collusion. With the same logic, total expected utilities of the supervisor and the principal when the supervisor actually monitors should be bigger than total expected utilities they can achieve by collusion. Since principals is paying for the wages, its expected cost should be written negatively. In normal monitoring case, principal's expected cost is $\mu[\pi(w_H + s_H) + (1-\pi)(w_L + s_L)] + (1-\mu)(w_\emptyset + s_\emptyset)$ and supervisor's expected utility is $\mu[\pi s_H + (1-\pi)s_L] + (1-\mu)(s_\emptyset) - c_m$. If they agree to collude, supervisor earns $s_\emptyset$ and principal pays $s_\emptyset + w_\emptyset$. Therefore, upward ex-ante collusion constraint is $\mu[\pi s_H + (1-\pi)s_L] + (1-\mu)(s_\emptyset) - c_m - (\mu[\pi(w_H + s_H) + (1-\pi)(w_L + s_L)] + (1-\mu)(w_\emptyset + s_\emptyset)) \geq s_\emptyset - (s_\emptyset + w_\emptyset)$ that can be simplified as

$$w_\emptyset \geq \frac{c_m}{\mu} + [\pi w_H + (1-\pi)w_L] \tag{7}$$

This new constraint brings in a restriction on agent's wages by introducing a lower bound on $w_\emptyset$. If $w_\emptyset$ is below the threshold given by the expression at the right hand side of (7), there would be a positive surplus from collusion between the supervisor and the principal. Note that the lower bound on $w_\emptyset$ depends on c_m because under collusion the principal would economize from wages to the supervisor who does not, accordingly, monitor. By adding (7) constraint into the objective function of the principal in 4.1, we will acquire optimal contract that accounts for ex-ante upward collusion.

Proposition 2

a. *The optimal ex-post collusion-proof and ex-ante downward collusion-proof contract (assuming ex-ante upward collusion is not possible) and the principal's corresponding expected cost of inducing high effort are:*
$(w_L^D, w_\emptyset^D, w_H^D) = (0, \frac{c_e}{\mu\pi}, \frac{c_e}{\mu\pi})$, $(s_L^D, s_\emptyset^D, s_H^D) = (\sigma_L, 0, \sigma_H)$ *such that* $\mu[\pi\sigma_H + (1-\pi)\sigma_L] = c_m + \frac{c_e}{\pi}$ *satisfying* $\sigma_L \geq \frac{c_e}{\mu\pi}$ *and* $EC^D = c_m + c_e\frac{(1+\mu\pi)}{\mu\pi}$.

b. *The optimal ex-post collusion-proof and ex-ante upward collusion-proof contract (assuming ex-ante downward collusion is not possible) and the principal's corresponding expected cost of inducing high effort are:*
(i) $(w_L^U, w_\emptyset^U, w_H^U) = (0, \frac{c_e}{\mu\pi}, \frac{c_e}{\mu\pi})$, $(s_L^U, s_\emptyset^U, s_H^U) = (\frac{c_e}{\mu\pi}, 0, 0)$ *and* $EC^U = \frac{c_e}{\mu\pi}$ *if* $\frac{(1-\pi)}{\pi}c_e \geq c_m$*;*
(ii) $(w_L^U, w_\emptyset^U, w_H^U) = (0, \frac{c_m+c_e}{\mu}, \frac{c_e}{\mu\pi})$, $(s_L^U, s_\emptyset^U, s_H^U) = (\sigma_L, 0, \sigma_H)$ *such that* $\mu[\pi\sigma_H + (1-\pi)\sigma_L] = c_m$ *satisfying* $s_L \geq \frac{c_m+c_e}{\mu}$ *and* $EC^U = \frac{c_m+c_e}{\mu}$*, if* $\frac{(1-\pi)}{\pi}c_e \leq c_m$.

Observe that introducing the ex-ante downward collusion constraint on top of ex-post collusion constraints did not have any effect on agent's wages. However, it suppressed supervisor's IC constraint and lead to an increase in supervisor's wages. Without any condition on c_m any other variable, this ex-ante constraint binds. To prevent ex-ante downward collusion we need to increase either s_L or s_H even further than the amount required to give supervisor monitoring incentive. While keeping s_L above $\frac{c_e}{\mu\pi}$ to satisfy (4), any combination of these wages satisfying $\mu[\pi\sigma_H + (1-\pi)\sigma_L] = c_m + \frac{c_e}{\pi}$ will be optimal for the principal.[6]

Even if it is compared to the worse case of ex-post protected institution that is *(ii)* stated in proposition 1, there is an increase of $\frac{c_e}{\pi}$ in expected cost for the principal. From this result, we conclude that, there is indeed a downward collusion possibility just after the contracts are proposed, so that, principal should take this threat into account while designing contracts.

For the contracts preventing ex-ante upward collusion, there are two distinguishing cases similar to the ex-post collusion-proof case. Condition for these cases are exactly the same but the contracts have differences. If c_m is below $\frac{(1-\pi)}{\pi}c_e$, then the contracts are same since ex-ante upward constraint does not bind and it has no effect. However, if $c_m \geq \frac{(1-\pi)}{\pi}c_e$ then principal need to increase $w_\emptyset$ to prevent the collusion. Increasing

[6]Specifically, when s_L is minimal and equal to $\frac{c_e}{\mu\pi}$ the principal sets $s_H = \frac{c_m}{\mu\pi} - \frac{(1-\mu-\pi)}{\mu\pi}\frac{c_e}{\mu\pi}$, whereas if $s_H = 0$ then s_L is maximal and equal to $\frac{\pi c_m + c_e}{\mu\pi(1-\pi)}$.

$w_\emptyset$ also increases the lower bound for s_L to satisfy ex-post downward collusion constraint. In this case, lower bound for s_L becomes $\frac{c_m+c_e}{\mu}$. It should be noted that, in this condition, supervisor's IC constraint always binds and the principal needs to increase s_L and s_H to give enough incentive to supervisor to monitor. [7]

Principal still set s_L and s_H combination to c_m. Therefore, there is no additional cost from there. On the other hand, to prevent this ex-ante upward collusion, there should be increase in $w_\emptyset$. Depending on the amount that c_m exceeds $\frac{(1-\pi)}{\pi}c_e$, $w_\emptyset$ has to be increased. At the end, this will result in increase in expected cost.

Until now, we have shown that both ex-ante constraints should be considered while contracts are designed. They are not implied by other constraints and there are actual collusion possibilities. In the next step, we will look for the optimal contracts that prevents all collusion threats.

4.2.2 Fully protected contracts

When the contract is full collusion-proof, none of the players in this hierarchy can benefit from bribing another. They will fulfil their duties: the agent will set $e = 1$ and the supervisor will monitor, $m = 1$. To achieve this outcome at minimum expected cost, the principal must solve the following problem:

$$\begin{array}{ll} \min\limits_{w_L,w_\emptyset,w_H,s_L,s_\emptyset,s_H} & \mu[\pi(w_H+s_H)+(1-\pi)(w_L+s_L)]+(1-\mu)(w_\emptyset+s_\emptyset) \\ \text{subject to} & (1),(2),(3),(4),(5),(6)\ \textit{and}\ (7) \end{array}$$

Proposition 3 *The optimal full collusion-proof contract and the principal's corresponding expected cost of inducing high effort are:*
(i) $(w_L, w_\emptyset, w_H) = (0, \frac{c_e}{\mu\pi}, \frac{c_e}{\mu\pi})$, $(s_L, s_\emptyset, s_H) = (\sigma_L, 0, \sigma_H)$ *such that* $\mu[\pi\sigma_H + (1-\pi)\sigma_L] = c_m + \frac{c_e}{\pi}$ *satisfying* $s_L \geq \frac{c_e}{\mu\pi}$ *and* $EC = c_m + c_e\frac{(1+\mu\pi)}{\mu\pi}$ *if* $\frac{(1-\pi)}{\pi}c_e \geq c_m$*;*
(ii) $(w_L, w_\emptyset, w_H) = (0, \frac{c_m+c_e}{\mu}, \frac{c_e}{\mu\pi})$, $(s_L, s_\emptyset, s_H) = (\sigma_L, 0, \sigma_H)$ *such that* $\mu[\pi\sigma_H + (1-\pi)\sigma_L] = 2c_m + c_e$ *satisfying* $s_L \geq \frac{c_m+c_e}{\mu}$ *and* $EC = c_m\frac{(1+\mu)}{\mu} + c_e\frac{(1+\mu)}{\mu}$ *if* $\frac{(1-\pi)}{\pi}c_e \leq c_m$.

There is a significant change in the wage structure compared to the ex-post collusion-proof contracts given in proposition 1. Both propositions 1 and 3 have two distinguishing cases and condition for these cases are same. For case (i), agent's wages do not change, but now combination of wages s_L and s_H should increase to claim full collusion-

[7] In detail, when s_L is minimal and equal to $\frac{c_m+c_e}{\mu}$ the principal sets $s_H = \frac{c_m}{\mu} - \frac{(1-\pi)}{\mu\pi}$, whereas if $s_H = 0$ then s_L is maximal and equal to $\frac{c_m}{\mu(1-\pi)}$.

proofness. Constraint on s_L does not change.[8] For case (ii), there is an increase in $w_\emptyset$ to prevent ex-ante upward collusion. Also, due to increase in the agent's wages, the supervisor's wages further increases.[9] Note that, expected cost for the principal also increases due to wage increases.

For both cases, the principal increases the combination of s_L and s_H wages above c_m to satisfy the ex-ante downward collusion constraint. Therefore, supervisor's IC is automatically satisfied. Both ex-post upward and ex-ante upward constraints put a restriction on $w_\emptyset$. Two constraints cannot bind at the same time, depending on c_e and c_m, binding constraint changes. If the supervision cost c_m is very low, ex-post upward constraint binds so that preventing ex-post upward collusion prevents ex-ante upward collusion threat as well. On the other hand, if c_m is higher than $\frac{(1-\pi)}{\pi}c_e$, preventing ex-ante upward collusion becomes more costly since the principal needs to pay premium to prevent possibility of collusion between himself and the principal to induce agent to put effort.

Moreover, the ex-post collusion constraints are always binding. The ex-post downward collusion constraint puts a lower bound on s_L whereas the ex-ante downward collusion constraint imposes a restriction on some combination of s_L and s_H. In total, when the supervisor conducts output monitoring, supervisor's IC and, depending on c_e and c_m, one of the upward collusion constraints becomes redundant.

[8]When s_L is minimal and equal to $\frac{c_e}{\mu\pi}$ the principal sets $\frac{c_m}{\mu\pi} - \frac{(1-\mu-\pi)}{\mu\pi}\frac{c_e}{\mu\pi}$, whereas if $s_H = 0$ then s_L is maximal and equal to $\frac{\pi c_m + c_e}{\mu\pi(1-\pi)}$.

[9]When s_L is minimal and equal to $\frac{c_m + c_e}{\mu}$ the principal sets $s_H = \frac{(1+\pi)c_m + \pi c_e}{\mu\pi}$, whereas if $s_H = 0$ then s_L is maximal and equal to $\frac{2c_m + c_e}{\mu(1-\pi)}$.

5 Should the principal prevent ex-post collusion?

After showing that the full collusion-proof contracts have significantly higher expected costs for the principal, we know look for the best strategy for principal to minimize this cost while inducing agent to put effort. The principal has to prevent both ex-ante collusions at any cost since high output has infinite value for the principal and only way to produce output is that the agent actually works. On the other hand, the principal can decide on whether ex-post collusion should be prevented or not. There exist four strategies which can be used: preventing both downward and upward collusion, permitting downward collusion, permitting upward collusion and permitting both types of collusions. Vafai showed that in the absence of ex-ante collusion, best strategy would be to prevent all ex-post collusions and offer a full collusion-proof contract. We know check whether his claim can be extended when ex-ante collusions are introduced to the hierarchy. To find out the answer of this question, we need to know the expected costs in each case. We already have optimal contracts for full collusion-proof case that is preventing both ex-post collusions strategy at the end of the previous chapter. We solve the principal's problem for remaining three other strategies and compare the results.

5.1 Permit downward ex-post collusion

All the constraints discussed in the previous sections are subject to change except for limited liabilities (1). When we allow for any type of ex-post collusion, parties will be aware of the situation and their incentive constraints will change. When the supervisor finds low output, the agent can offer bribe up to $b_{\mathrm{DC}} = w_\emptyset - w_L$ to the supervisor, make him reveal empty output. Since we assumed the supervisor does not engage in corruption when indifferent, bribe should be strictly positive and as small as possible.

$$w_\emptyset - w_L \geq k. \tag{8}$$

Note that, in this constraint $k \geq 0$ and $k \to 0$.

We wanted to permit downward ex-post collusion which requires $s_L < s_\emptyset + (w_\emptyset - w_L)$. To get rid of strict inequality we use the surplus coming from bribe, and the constraint becomes

$$s_L \leq s_\emptyset + (w_\emptyset - w_L - k). \tag{9}$$

We still prevent upward ex-post collusion. Thus, we borrow this constraint directly (5).

We need to redefine incentive compatibility constraints and ex-ante collusion-proofness constraints. Agent now gives a bribe to the supervisor if she finds a low output. We can write agent's IC as $\mu[\pi w_H + (1-\pi)(w_\emptyset - b_{\mathrm{DC}})] + (1-\mu)w_\emptyset - c_e \geq \mu(w_\emptyset - b_{\mathrm{DC}}) + (1-\mu)w_\emptyset$. Since the agent offers all the surplus that will come from deviation to the empty report, his incentive constraint will be same as it is in the full collusion proof case that is

$$w_H - w_L \geq \frac{c_e}{\mu\pi}. \tag{10}$$

Since the agent has to offer all the surplus coming from difference between the empty report wage and low output wage, his incentives do not change when ex-post downward collusion is permitted.

Supervisor will accept the bribe, when she finds low output, she will get $s_\emptyset + b_{\mathrm{DC}}$ instead of s_L. In total supervisor's IC is $\mu[\pi s_H + (1-\pi)(s_emptyset + w_\emptyset - w_L)] + (1-\mu)s_\emptyset - c_m \geq s_\emptyset$. Simplification yields

$$\mu\pi s_H + w_\emptyset(\mu - \mu\pi) \geq c_m + w_L(\mu - \mu\pi) + \mu\pi s_\emptyset. \tag{11}$$

Notice that her incentive does not include s_L wage anymore since it will be already offered as bribe if she can find a proof of low output.

We are done with ex-post collusion constraint and incentive compatibility constraints. Now, we need to write down ex-ante collusion-proofness constraints and minimize the expected cost of the principal in this environment.

Ex-ante downward collusion constraint is $\mu[\pi s_H + (1-\pi)(s_\emptyset + w_\emptyset - w_L] + (1-\mu)s_\emptyset - c_m + \mu[\pi s_H + (1-\pi)w_L] + (1-\mu)w_\emptyset - c_e \geq s_\emptyset + w_\emptyset$. Simplified version is

$$\mu\pi w_H + \mu\pi s_H \geq c_m + c_e + \mu\pi s_\emptyset + \mu\pi w_\emptyset. \tag{12}$$

When ex-post downward collusion is permitted, increasing low output wages does not help us to prevent ex-ante downward collusion as it was in the full-collusion proof case.

Ex-ante upward collusion-proofness constraint is $\mu[\pi s_H + (1-\pi)(s_\emptyset + w_\emptyset - w_L)] + (1-\mu)(s_\emptyset) - c_m - (\mu[\pi(w_\emptyset + s_\emptyset + w_H + s_H - w_\emptyset - s_\emptyset) + (1-\pi)(w_\emptyset + s_\emptyset)] + (1-\mu)(w_\emptyset + s_\emptyset)) \geq s_\emptyset - (s_\emptyset + w_\emptyset)$. It can be simplified as

$$w_\emptyset \geq \frac{c_m}{\mu} + [\pi w_H + (1-\pi)w_L]. \tag{13}$$

This constraint also did not change when we permit ex-post downward collusion.

Below we produce the objective function of the principal in this environment. The solution to this problem will deliver us the optimal contracts that only allow ex-post downward collusion; all other collusion possibilities are prevented.

$$\begin{array}{ll} \min\limits_{w_L, w_\emptyset, w_H, s_L, s_\emptyset, s_H} & \mu\pi(w_H + s_H) + (1-\mu\pi)(w_\emptyset + s_\emptyset) \\ \text{subject to} & (1), (8), (9), (5), (10), (11), (12) \; and \, (13) \end{array}$$

Proposition 4 *Ignore the possibility of downward ex-post collusion. The optimal ex-ante collusion-proof contract and the principal's corresponding expected cost of inducing high effort are:*
(i) $(w_L, w_\emptyset, w_H) = (0, \frac{c_e}{\mu\pi}, \frac{c_e}{\mu\pi})$, $(s_L, s_\emptyset, s_H) = (0, 0, \frac{c_m+c_e}{\mu\pi})$ *and* $EC = c_m + c_e\frac{(1+\mu\pi)}{\mu\pi}$ *if* $\frac{(1-\pi)}{\pi}c_e \geq c_m$;
(ii) $(w_L, w_\emptyset, w_H) = (0, \frac{c_m+c_e}{\mu}, \frac{c_e}{\mu\pi})$, $(s_L, s_\emptyset, s_H) = (0, 0, c_m\frac{(1+\pi)}{\mu\pi} + \frac{c_e}{\mu})$ *and* $EC = c_m\frac{(1+\mu)}{\mu} + c_e\frac{1+\mu}{\mu}$ *if* $\frac{(1-\pi)}{\pi}c_e \leq c_m$.

In the case of full collusion proof implementation, the principal expects to pay $w_\emptyset$ with probability $(1-\mu)$. When ex-post downward collusion is permitted, this probability increases to $(1-\mu\pi)$, because the principal knows that low output will be reported as an empty report. In view of this fact, the probability of paying low output wages is zero. Also, weight of s_H and w_H in the expected cost function of the principal do not change because there is no upward collusion.

However, these changes have no impact on expected costs. The principal adjusts all the wages according to the new revealing likelihoods of each type of reports and satisfy all constraints except the ex-post downward collusion constraint. To prevent ex-ante downward collusion, the principal was offering a combination of s_L and s_H wages; now he offers only s_H. Also, as weights of wages in expected cost changed and since $w_\emptyset$ is more likely to be paid in this case, effect of s_H in expected cost will be lower compared to combination of s_L and s_H in full collusion-proof case.

In sum, the possibility of a low output report is taken out of the equation, so, the principal offers s_H instead of s_L, which does not bring in any extra cost because the parties are allowed to collude ex-post. In other words, the principal successfully induces the agent to exert effort and downward collusion, which means that side transfers between these two parties cancel out from the principal's cost objective.

5.2 Permit upward ex-post collusion

We will redefine the constraints which guarantee upward ex-post collusion. If the supervisor reveals high output, the principal approaches to the supervisor and offers a bribe to change the high output report to the empty report. Maximum amount of bribe that can be offered is $b_{\mathrm{UC}} = w_H + s_H - w_\emptyset - s_\emptyset$. Using a similar logic, we write the following constraint. As defined, $k \geq 0$ and $k \to 0$.

$$w_H + s_H - w_\emptyset - s_\emptyset \geq k \tag{14}$$

We wanted to permit upward collusion, so related constraint, ex-post upward collusion constraint, should be inverted. By doing this, we make sure that the supervisor and the principal benefit from upward collusion.

$$w_H \geq w_\emptyset + k. \tag{15}$$

We need to prevent upward ex-post collusion. Related constraint (5) has been already defined at the beginning of the previous chapter, we directly use it without any modification.

Agent's IC will be $\mu[\pi w_\emptyset + (1-\pi)w_L] + (1-\mu)w_\emptyset - c_e \geq \mu w_L + (1-\mu)w_\emptyset$. After simplification,

$$w_\emptyset - w_L \geq \frac{c_e}{\mu\pi}. \tag{16}$$

When ex-post upward collusion is allowed, the agent will know that the supervisor does not reveal high output even though she obtains hard evidence of high output, because she will collude with the principal. The report will be empty. As a result, the agent's incentive to exert high effort now depends on the wage difference between $w_\emptyset$ and w_L instead of w_H and w_L.

Supervisor's IC is $\mu[\pi(s_\emptyset + w_H + s_H - w_\emptyset - s_\emptyset) + (1-\pi)s_L] + (1-\mu)s_\emptyset - c_m \geq s_\emptyset$ which is simplified as

$$\mu\pi s_H + s_L(\mu - \mu\pi) + \mu\pi w_H \geq c_m + \mu\pi w_\emptyset + \mu s_\emptyset. \tag{17}$$

Now, w_H appears in the LHS of the supervisor's IC constraint because in the case of ex-post upward collusion the supervisor takes her bribe from this wage of the agent.

Ex-ante downward collusion constraint is $\mu[\pi(s_\emptyset + w_H + s_H - w_\emptyset - s_\emptyset) + (1-\pi)s_L] + (1-\mu)s_\emptyset - c_m + \mu[\pi w_\emptyset + (1-\pi)w_L] + (1-\mu)w_\emptyset - c_e \geq s_\emptyset + w_\emptyset$ and that is

$$\mu[\pi s_H + (1-\pi)s_L] + \mu[\pi w_H + (1-\pi)w_L] \geq \mu s_\emptyset + \mu w_\emptyset + c_m + c_e. \tag{18}$$

This constraint does not change when we allow for upward ex-post collusion.

Lastly, ex-ante upward collusion constraint is $\mu[\pi(s_\emptyset + w_H + s_H - w_\emptyset - s_\emptyset) + (1 -$

$\pi)(s_L)] + (1-\mu)(s_\emptyset) - c_m - (\mu[\pi(w_\emptyset + s_\emptyset + w_H + s_H - w_\emptyset - s_\emptyset) + (1-\pi)(w_L + s_L)] + (1-\mu)(w_\emptyset + s_\emptyset)) \geq s_\emptyset - (s_\emptyset + w_\emptyset)$.

$$w_\emptyset \geq \frac{c_m}{(\mu - \mu\pi)} + w_L. \tag{19}$$

We do not have w_H at the RHS when ex-post upward collusion is permitted because the supervisor already achieves to get $w_H - w_\emptyset$ ex-post.

$$\begin{aligned} \min_{w_L, w_\emptyset, w_H, s_L, s_\emptyset, s_H} \quad & \mu[\pi(w_H + s_H) + (1-\pi)(w_L + s_L)] + (1-\mu)(w_\emptyset + s_\emptyset) \\ \text{subject to} \quad & (1), (14), (15), (4), (16), (17), (18) \textit{ and } (19) \end{aligned}$$

Since the principal offers all the surplus as bribe, the objective function does not change.

Proposition 5 *Ignore the possibility of upward ex-post collusion. The optimal ex-ante collusion-proof contract and the principal's corresponding expected cost of inducing high effort are:*
(i) $(w_L, w_\emptyset, w_H) = (0, \frac{c_e}{\mu\pi}, \sigma_a)$, $(s_L, s_\emptyset, s_H) = (\sigma_b, 0, \sigma_c)$ *such that* $\mu\pi\sigma_a + (\mu - \mu\pi)\sigma_b + \mu\pi\sigma_c = c_m + c_e + \frac{c_e}{\pi}$ *satisfying* $\sigma_a \geq \frac{c_e}{\mu\pi} + k$, $\sigma_b \geq \frac{c_e}{\mu\pi}$ *and* $EC = c_m + c_e\frac{(1+\mu\pi)}{\mu\pi}$ *if* $\frac{(1-\pi)}{\pi}c_e \geq c_m$*;*
(ii) $(w_L, w_\emptyset, w_H) = (0, \frac{c_m}{(\mu-\mu\pi)}, \sigma_a)$, $(s_L, s_\emptyset, s_H) = (\sigma_b, 0, \sigma_c)$ *such that* $\mu\pi\sigma_a + (\mu - \mu\pi)\sigma_b + \mu\pi\sigma_c = c_m + c_e + \frac{c_m}{1-\pi}$ *satisfying* $\sigma_a \geq \frac{c_m}{\mu-\mu\pi} + k$, $\sigma_b \geq \frac{c_m}{\mu-\mu\pi}$ *and* $EC = c_m + c_e + \frac{c_m}{\mu-\mu\pi}$ *if* $\frac{(1-\pi)}{\pi}c_e \geq c_m$.

The objective function of the principal is exactly the same as in the case of full collusion-proof implementation because under ex-post upward collusion, the principal has to offer as bribe the whole surplus from an empty report instead of the surplus from a high output report. Thus, the principal will consider high output report wages while calcullating expected costs which results in same objective function as in full collusion proof case.

While we analyse this proposition, we need to differentiate between two cases depending on effort cost, monitoring cost and inefficiency parameter of the effort. We can directly mention each cases through only monitoring cost since it makes sense to conduct analysis over supervisor's incentives. Thus, in case *(i)* we have very low monitoring cost, in other case we have high monitoring costs.

For case *(i)*, we have same expected costs, permitting ex-post upward collusion does not result in any problem for the principal if the monitoring cost is low enough. For full collusion proof case, w_H was giving incentive to the agent not to engage in ex-ante downward collusion, now it gives incentive to the supervisor. However, in total, ex-ante downward collusion constraint is satisfied without harming the principal. Only the agent is harmed, and some expected income is transferred to the supervisor. The principal remains unaffected.

The critical point is that the ex-ante upward collusion constraint is automatically satisfied under a low monitoring cost, so that the effect of this constraint on $w_{\emptyset}$ can be ignored. While the principal can satisfy the agent's IC by increasing the difference between $w_{\emptyset}$ and w_L, he also prevents ex-ante collusion between the supervisor and the principal. In other words, when the supervisor faces low monitoring costs, she would find it beneficial to obtain an evidence first and then expect to be approached by the principal for collusion.

For case *(ii)* where the monitoring costs are high, expected costs are high, so, permitting ex-post upward collusion is not a good strategy for the principal.

Overall, for first case, we end up with same expected costs but for second case, permitting ex-post collusions is more costly for the principal. Difference between these two cases emerges solely due to resulting ex-ante upward collusion constraints. When ex-post upward collusion is prevented, ex-ante upward collusion is adjusted accordingly and it does not include w_H in RHS anymore. As a result of this $w_{\emptyset}$ does only depend on c_m instead of combination of c_m and c_e. Specifically, $\frac{c_e}{\mu}$ term in $w_{\emptyset}$ was due to w_H but permitting ex-post upward collusion has taken w_H out of the equation.

Since in case *(ii)*, we have high monitoring cost and ex-post upward collusion permitted contracts puts more weight on c_m in $w_{\emptyset}$, one unit increment of c_m costs more to the principal if ex-post upward collusion is permitted. In other words, now, more $w_{\emptyset}$ has to be offered to give enough incentive to the supervisor for monitoring, otherwise, ex-ante upward collusion occurs.

5.3 Permit both ex-post collusions

Now we can borrow positive bribe constraints (8), (14) necessary to deviate the supervisor since she does not engage in collusion when indifferent and inducing ex-post collusion constraints (9), (15) but we still need redefine incentive and ex-ante collusion constraints.

In this environment, agent's IC is $\mu[\pi w_{\emptyset}+(1-\pi)(w_{\emptyset}-b_{\text{DC}})]+(1-\mu)w_{\emptyset}-c_e \geq \mu(w_{\emptyset}-b_{\text{DC}}) + (1-\mu)w_{\emptyset}$. When the true output is high, the agent earns empty output wage which is a loss for the agent but he may also compensate this loss if the output is low by bribing the supervisor. Incorporate b_{DC} and simplify, agent's incentive compatibility constraint is

$$w_\emptyset - w_L \geq \frac{c_e}{\mu\pi}. \tag{20}$$

Supervisor will benefit from both collusions since he holds the information which is power in this context due to information asymmetry. Her incentive constraint without simplification is $\mu[\pi(s_\emptyset+w_H+s_H-w_\emptyset-s_\emptyset)+(1-\pi)(s_\emptyset+w_\emptyset-w_L)]+(1-\mu)s_\emptyset-c_m \geq s_\emptyset$. Cancel out terms, and group the same wages, we have

$$\mu\pi s_H + \mu\pi w_H \geq \mu\pi s_\emptyset + c_m + w_L(\mu - \mu\pi) + w_\emptyset(2\mu\pi - \mu). \tag{21}$$

In full collusion proof case, the supervisor's incentive was depending on s_L and s_H wages. She was motivated to find an evidence for the agent's output. However, once ex-post collusion possibilities are allowed, s_L did not mean anything to the supervisor because nevertheless she will achieve $s_\emptyset$ plus a bribe by colluding with the agent. Other than this, now she will consider the bribe $w_H - w_\emptyset$ that will be offered by the principal to reveal empty report instead of high output report in her expected utility. When upward collusion occurs, she takes this bribe but she does not give up $s_H - s_\emptyset$; this will be offered in addition to the bribe. In total, s_H and w_H become the main determinant for the supervisor's motivation.

Furthermore, ex-ante collusion constraints change, because both types of ex-post collusion are induced, which will be reflected in the parties' expected utilities. When the agent puts effort and the supervisor monitors, their total utility will be $\mu[\pi(s_\emptyset + w_H + s_H - w_\emptyset - s_\emptyset) + (1-\pi)(s_\emptyset + w_\emptyset - w_L)] + (1-\mu)s_\emptyset - c_m + \mu[\pi w_\emptyset + (1-\pi)(w_\emptyset - w_\emptyset + w_L)] + (1-\mu)w_\emptyset - c_e$. We adjusted expected wages with given bribes according to deviations to corrupted output states. This expected utility should be bigger than ex-ante downward collusion pay-offs. RHS of the equation is same, if they agree to collude ex-ante, they get $s_\emptyset + w_\emptyset$. Simplifying results in

$$\mu\pi s_H + \mu\pi w_H \geq c_m + c_e + \mu\pi w_\emptyset + \mu\pi s_\emptyset. \tag{22}$$

Lastly, we construct ex-ante upward collusion constraint. We want a state where the principal and the supervisor are not better off when they collude before the agent and the supervisor do their respective tasks. It happens when $\mu[\pi(s_\emptyset + w_H + s_H - w_\emptyset - s_\emptyset) + (1-\pi)(s_\emptyset + w_\emptyset - w_L)] + (1-\mu)s_\emptyset - c_m - (\mu[\pi(w_\emptyset + s_\emptyset + w_H + s_H - w_\emptyset - s_\emptyset) + (1-\pi)(w_\emptyset + s_\emptyset)] + (1-\mu)(w_\emptyset + s_\emptyset)) \geq s_\emptyset - s_\emptyset - w_\emptyset$. Simplification gives

$$w_\emptyset \geq \frac{c_m}{(\mu - \mu\pi)} + w_L. \tag{23}$$

The constraints permitting ex-post collusion are produced. We now solve the principal's adjusted cost minimization problem subject to all of these constraints combined. Notice that the objective function is the same as in the case of permitting only ex-post downward collusion. The problem is:

$$\min_{w_L,w_\emptyset,w_H,s_L,s_\emptyset,s_H} \quad \mu\pi(w_H+s_H)+(1-\mu\pi)(w_\emptyset+s_\emptyset)$$
$$\text{subject to} \quad (1),(8),(9),(14),(15),(20),(21),(22)\ \textit{and}\ (23)$$

Proposition 6 *Ignore the possibility of both ex-post collusions. The optimal ex-ante collusion-proof contract and the principal's corresponding expected cost of inducing high effort are:*
(i) $(w_L,w_\emptyset,w_H)=(0,\frac{c_e}{\mu\pi},\sigma_w)$, $(s_L,s_\emptyset,s_H)=(0,0,\sigma_s)$ *such that* $\sigma_w+\sigma_s=\frac{c_m+2c_e}{\mu\pi}$ *satisfying* $\sigma_w\geq\frac{c_e}{\mu\pi}+k$ *and* $EC=c_m+c_e\frac{(1+\mu\pi)}{\mu\pi}$ *if* $\frac{(1-\pi)}{\pi}c_e\geq c_m$*;*
(ii) $(w_L,w_\emptyset,w_H)=(0,\frac{c_m}{\mu-\mu\pi},\sigma_w)$, $(s_L,s_\emptyset,s_H)=(0,0,\sigma_s)$ *such that* $\sigma_w+\sigma_s=\frac{c_m}{\mu\pi(1-\pi)}+\frac{c_e}{\mu\pi}$ *satisfying* $\sigma_w\geq\frac{c_m}{\mu-\mu\pi}+k$ *and* $EC=c_m+c_e+\frac{c_m}{\mu-\mu\pi}$ *if* $\frac{(1-\pi)}{\pi}c_e\leq c_m$*.*

In these contracts, we see the effects of permitting both ex-post collusions together. $s_L=0$ due to ex-post downward collusion and $w_\emptyset$ is the main actor shaping the structure of the contracts through ex-post upward collusion.

For case *(i)*, we have same expected costs and for case *(ii)*, we have higher expected costs which is equal to the case where we only permit ex-post upward collusion. Thus, ultimately, we can claim that permitting ex-post upward collusion increases the expected costs and the principal should prevent it. This raise in expected costs are because of ex-ante upward collusion constraint. Since the supervisor calculates her payoff from trustworthy reporting by including bribe that will be offered if she finds an high output, it becomes harder to keep her monitoring the agent without colluding ex-ante with the principal.

Corollary 1 *Even after ex-ante collusion constraints are introduced to the hierarchy, preventing all types of collusions remains as the optimal strategy.*

We have shown that optimal strategy of the principal does not change by comparing each of the other three strategies as an alternative. We infer that introducing supervision cost brought out ex-ante collusion possibilities but it did not have any effect on optimal strategy. The principal still finds preventing all types of collusions efficient.

When ex-post downward collusion is allowed, wage structure has slightly changed but their weights of the wages in the expected cost function were adjusted as well. At the end, it did not raised the expected costs. Side transfers between the agent and the supervisor does not relate to principal's expected costs. We can only infer that the money will be transferred to the supervisor from the agent but still the hierarchy works as the principal desired.

On the other hand, when ex-post upward collusion is permitted, the principal faces with a different situation. Objective function does not change so that each wage is expected to be paid with same probability as it in full collusion proof case. However, ex-ante upward collusion constraint is modified as the supervisor accounts for the bribe she will get by colluding with the principal ex-post. The supervisor now compares her monitoring cost with $w_\emptyset$ to decide whether to collude with the principal ex ante or not. In full collusion proof case, she was also considering w_H in her expected utility now she ignores since the principal offers $w_H - w_\emptyset$ already as a bribe for ex-post upward collusion. w_H was the reason we see $\frac{c_e}{\mu}$ term in $w_\emptyset$. At present case, $w_\emptyset = \frac{c_m}{(\mu - \mu\pi)}$, so that c_m gets more weight in this wage. Ultimately, we are in case where $\frac{(1-\pi)}{\pi} c_e \leq c_m$. As a result of this, for case *(ii)* where ex-ante upward collusion constraint determines the $w_\emptyset$, the principal has higher expected costs. We can also directly show $\frac{c_m}{\mu - \mu\pi} \geq \frac{c_m + c_e}{\mu}$ meaning that when ex-post upward collusion is permitted, preventing ex-ante upward collusion is harder. This increase in $w_\emptyset$ leads to raises in other wages as well. In total, when c_m is above a level, full collusion proof contracts become weakly favourable for the principal.

After showing that designing full-collusion proof contract is weakly optimal strategy for the principal, in the next section, we now turn to the analysis of a new type of monitoring called input monitoring.

6 Supervisor monitors input (effort)

In this section, we derive the optimal contracts when the supervisor monitors the agent's effort input only. For a meaningful comparison with the output monitoring case, the critical point is the technology of effort inspection. We shall maintain the structure of the monitoring technology used in the output monitoring case.

Because output is no longer monitored, the probability π that links the agent's effort to output becomes irrelevant. The constraints to the principal's problem will, accordingly, change. The supervisor obtains hard evidence of effort with probability μ, which she can submit in her report to the principal or conceal it and submit an empty report. With probability $1-\mu$ effort inspection fails and the supervisor's report is empty. The analysis below follows the same order as in the analysis of contracts under output monitoring.

6.1 Optimal ex-post collusion-proof contracts

The limited liability constraints will remain as in (1); the wages paid to the supervisor and the agent cannot be negative as in the output monitoring case. However, whereas the agent was taking into account the possibility of a low output under high effort, now the only relevant issue is whether effort inspection will succeed or fail. Thus, the agent's incentive compatibility constraint in (2) will be modified as $\mu w_H + (1-\mu)w_\emptyset - c_e \geq \mu w_L + (1-\mu)w_\emptyset$, which can be simplified to

$$w_H - w_L \geq \frac{c_e}{\mu}. \tag{24}$$

This inequality is similar to its counterpart in the output monitoring case with the exception that the denominator at the right hand side does not contain π.

Similarly, assuming that the agent exerts high effort, the supervisor's incentive compatibility constraint can be written as $\mu s_H + (1-\mu)s_\emptyset - c_m \geq s_\emptyset$, which simplifies to

$$s_H - s_\emptyset \geq \frac{c_m}{\mu}. \tag{25}$$

As in the case of output monitoring, the power of supervision incentives can be raised by increasing s_L and/or s_H, the wages in the outcomes that can arise only if the supervisor inspects. The important difference from output monitoring case is that a low-effort report is impossible under high effort, whereas a low output report was possible under high effort. Now, provided the agent's incentive compatibility constraint is satisfied, the agent will put high effort and there is no chance that monitoring can detect low effort level. Therefore, when the agent is induced to exert high effort the supervisor's incentive compatibility constraint in (25) can be satisfied by increasing s_H only, or by decreasing $s_\emptyset$ if possible. Observe that the incentive compatibility constraints above are formulated under the assumption that the parties do not collude, ex-ante or ex-post.

The occasions for ex-post collusion are identical. When the supervisor obtains effort evidence, she should not, along with the principal or the agent, strike a deal to submit an empty report instead of truthfully reporting the effort evidence. Thus, ex-post downward (4) and upward collusion-proof (5) constraints do not change. Combining the incentive compatibility constraints with limited liability and ex-post collusion-proofness constraints, the principal's problem can be written as

$$\begin{aligned} \min_{w_L, w_\emptyset, w_H, s_L, s_\emptyset, s_H} \quad & \mu(w_H + s_H) + (1-\mu)(w_\emptyset + s_\emptyset) \\ \text{subject to} \quad & (1), (24), (25), (4) \ \textit{and} \ (5) \end{aligned}$$

Note that the expression of the expected wage bill (the objective function) also changes along with the constraints. The solution to principal's problem is given below.

Proposition 7 *Suppose that ex-ante collusions are not possible. Optimal ex-post collusion-proof contracts and the principal's expected cost of inducing high effort by the agent are:* $(w_L^X, w_\emptyset^X, w_H^X) = (0, \frac{c_e}{\mu}, \frac{c_e}{\mu})$, $(s_L^X, s_\emptyset^X, s_H^X) = (\frac{c_e}{\mu}, 0, \frac{c_m}{\mu})$ *and* $EC^X = c_m + \frac{c_e}{\mu}$.

Although the principal does not actually pay the wages corresponding to low effort, he has to adjust the s_L wage to prevent ex-post downward collusion.

Ignoring ex-ante collusion possibilities, the ex-post collusion-proof contracts are described in Proposition 1 (output monitoring) and Proposition 7 (input monitoring). The differences in these propositions are purely due to the difference in the monitoring methods. Regardless of the monitoring technology, the principal sets $s_\emptyset = 0$ since increasing this wage does not provide any benefit but increases the cost of collusion prevention, by incentivizing the supervisor to deviate to submitting an empty report and get the higher wage $s_\emptyset$. Thus, given $s_\emptyset = 0$, observe that the ex-post downward collusion constraint brings a lower bound for s_L. In the output monitoring case, the supervisor's monitoring incentive depends on her wages under reports of low and high output. Thus, if monitoring cost is below a level, the principal does not need to assign any wage to high output, s_L wage is sufficient enough to motivate the supervisor for monitoring. However, in the input monitoring case, the supervisor knows that she does not get s_L since agent's IC constraint will already be satisfied, so the agent's effort level cannot be zero, leading to high output. It means that, the supervisor's monitoring incentive only depends on s_H. The amount of this wage will directly depend on monitoring cost and monitoring efficiency.

6.2 Optimal full collusion-proof contracts

In this section, the effects of ex-ante downward and ex-ante upward collusion constraints will be analysed separately to observe their individual impacts on optimal contracts and expected costs. Note that the ex-ante collusion constraints have to be redefined as it is done for incentive constraints since they include π variable.

6.2.1 Ex-ante downward and ex-ante upward protected contracts

Ex-ante downward constraint without any simplification is $\mu s_H + (1-\mu)s_\emptyset - c_m + \mu w_H + (1-\mu)w_\emptyset - c_e \geq s_\emptyset + w_\emptyset$. The right hand side, $s_\emptyset + w_\emptyset$, represents the total utilities of the agent-supervisor pair (the supervisor by not inspecting and the agent by exerting low effort guarantee the wages under an empty report). Simplifying, this constraint becomes

$$s_H + w_H \geq \frac{c_e + c_m}{\mu} + s_\emptyset + w_\emptyset. \tag{26}$$

The principal and the supervisor can collude before the agent decides on his effort and the supervisor makes her monitoring decision. To prevent this, we need $\mu s_H + (1-\mu)s_\emptyset - c_m - [\mu(w_H + s_H) + (1-\mu)(w_\emptyset + s_\emptyset) \geq s_\emptyset - (s_\emptyset + w_\emptyset)$. Simplified version is

$$w_\emptyset \geq w_H + \frac{c_m}{\mu}. \tag{27}$$

We will consider the effects of ex-ante constraints on optimal contracts separately by adding (26) and (27) as additional constraint to EC function in (6.1). When we analyse one one constraint we ignore the other collusion possibility.

Proposition 8

a. *The optimal ex-post collusion-proof and ex-ante downward collusion-proof contract (assuming ex-ante upward collusion is not possible) and the principal's expected cost of inducing high effort are:*
$(w_L^D, w_\emptyset^D, w_H^D) = (0, \frac{c_e}{\mu}, \frac{c_e}{\mu})$, $(s_L^D, s_\emptyset^D, s_H^D) = (\frac{c_e}{\mu}, 0, \frac{c_m + c_e}{\mu})$ *and* $EC^D = c_m + c_e\frac{(1+\mu)}{\mu}$.

b. *The optimal ex-post collusion-proof and ex-ante upward collusion-proof contract (assuming ex-ante downward collusion is not possible) and the principal's expected cost*

of inducing high effort are:
$(w_L^U, w_\emptyset^U, w_H^U) = (0, \frac{c_m+c_e}{\mu}, \frac{c_e}{\mu})$, $(s_L^U, s_\emptyset^U, s_H^U) = (\frac{c_m+c_e}{\mu}, 0, \frac{c_m}{\mu})$ *and* $EC^U = \frac{c_m+c_e}{\mu}$.

In the case of input monitoring, the role of the ex-ante downward collusion constraint is confined to raising s_H; all other wages are the same. The LHS of the ex-ante downward constraint represents the total utility from trustworthy reporting. The wages paid for low output cancel out from the equations. As a result, instead of combinations of s_L and s_H, only the high output wage s_H becomes relevant. We conclude that when the supervisor monitors the effort level of the agent, the ex-ante downward collusion constraint is binding and the threat of ex-ante downward collusion is credible.

Comparing proposition 4 and 5.b gives us the effect of introducing the possibility of ex-ante upward collusion on the contracts. We see that increasing the agent's wage $w_\emptyset$ helps to eliminate the risk of ex-ante upward collusion because under collusion the supervisor's report will be empty, so, to eliminate his incentive to collude with the supervisor, the principal has to penalize himself ex-ante with a higher wage payment to the agent under an empty report. Note that ex-post upward collusion constraint is implied by ex-ante upward collusion constraint. Both of these constraints put a bound on $w_\emptyset$ but ex-ante upward collusion constraint is binding since when m=0, the principal has to pay an additional $\frac{c_m}{m}$ wage to the empty report wage of the agent. We account for the excess utility resulting from not monitoring the agent.

6.2.2 Fully protected contracts

This section presents the optimal full-collusion proof contracts under input monitoring. In the previous two sections, we have shown that each ex-ante collusion constraint is binding when introduced separately. Now, we will be able to observe exact differences between ex-post collusion proof contracts and full-collusion proof contracts.

$$\begin{array}{ll} \min\limits_{w_L, w_\emptyset, w_H, s_L, s_\emptyset, s_H} & \mu(w_H + s_H) + (1-\mu)(w_\emptyset + s_\emptyset) \\ \text{subject to} & (1), (24), (25), (4), (5), (26) \text{ } and \text{ } (27) \end{array}$$

Proposition 9 *The optimal full collusion-proof contract and the principal's corresponding expected cost of inducing high effort are:*
$(w_L, w_\emptyset, w_H) = (0, \frac{c_m+c_e}{\mu}, \frac{c_e}{\mu})$, $(s_L, s_\emptyset, s_H) = (\frac{c_m+c_e}{\mu}, 0, \frac{2c_m+c_e}{\mu})$ *and* $EC = c_m\frac{(1+\mu)}{\mu} + c_e\frac{(1+\mu)}{\mu}$.

An obvious implication of this proposition is that regardless of the monitoring method, moving from ex-post-only collusion-proof contracts to full-collusion proof contracts increases the expected cost for the principal significantly. At most, if $\mu = 1$, the expected costs are exactly doubled, raising from $c_e + c_m$ to $2c_e + 2c_m$. But this cost increase must be incurred to ensure that collusion possibilities are eliminated so that the agent exerts effort.

When we consider the impact of introducing both types of ex-ante collusion possibilities at the same time, we observe their cumulative impact on optimal contracts. Agent's empty report wage increases from $\frac{c_e}{\mu}$ to $\frac{c_m+c_e}{\mu}$ because of ex-ante upward collusion constraint. This change in $w_\emptyset$ leads to increase in both low and high output wages of the supervisor. First, it increases s_L through ex-post downward collusion constraint. Raised $w_\emptyset$ strengthens agent's hand in ex-post downward collusion deal. To prevent this, the principal needs to offer more to the supervisor to motivate him for revealing low output. As we have shown in proposition 5.a, ex-ante downward collusion constraint is binding. Now that we increased $w_\emptyset$ and it is in RHS of the ex-ante downward collusion constraint, s_H needs to be increased further.

Supervisor's IC constraint is suppressed by downward ex-ante collusion proof constraint as it is in the output monitoring. While the principal tries to prevent ex-ante downward collusion, s_H is set high enough that automatically gives the supervisor monitoring incentive. Again, both ex-post upward and ex-ante upward constraints put a restriction on $w_\emptyset$ but this time, ex-ante upward constraint directly puts a bigger restriction. Another difference is that, when the input is monitored, w_L term in the ex-ante downward constraint disappears, the principal does not have to pay to the agent when the output is low.

Preventing ex-ante collusions requires to take effort and monitoring costs into account so that intuitively ex-ante collusion proof contracts might be full collusion proof as well. However, we found no evidence for this claim, ex-post downward constraint is binding independently. Detailed analysis of monitoring types, effect of new introduced ex-ante collusion constraints are given in the next section.

7 Discussion of the results

In chapter 4 and chapter 6, we have introduced ex-ante collusion-proof constraints and showed their effect on optimal contracts. For both output and input monitoring, we have seen that additional ex-ante downward collusion constraint lead to an increase in s_L and s_H through various mechanisms. Also, ex-ante upward collusion constraint increased $w_\emptyset$ which leads to indirect increase in supervisor wages. Combining the propositions so far yields the following result, which holds regardless of whether the supervisor is instructed to monitor the output or the input.

Corollary 2 *Supervisor's monitoring incentive is automatically satisfied when contracts are designed full collusion proof.*

The principal does not need to consider the supervisor's incentive compatibility constraint since it is not binding. While ex-ante downward collusion is prevented through increase in s_L and s_H, more than enough incentive is given to the supervisor to monitor. Ex-ante collusion constraints include monitoring cost and effort cost as well as deviations to corrupted output. It is quite logical to achieve this result. More interestingly, same conclusion does not hold for the agent's incentive compatibility due to inefficiency of the effort. Unlike supervisor's monitoring, agent's effort creates randomness in the output, thus we still need to adjust agent's incentive.

Corollary 3 *Regardless of type of monitoring, all four collusion constraints cannot bind at the same time.*

Although there same some links between ex-ante collusion and ex-post collusion, preventing one type of collusion does not prevent the other. Specifically, both ex-post downward and ex-ante downward collusion constraints are binding regardless of type of monitoring. For both monitoring types, ex-post downward collusion constraint puts a lower bound on s_L. On the other hand, ex-ante downward collusion constraint restricts combination of s_L and s_H for the output monitoring and it puts a lower bound for s_H when the input is monitored. Since these constraints determine the lower bounds of different wages or some combination of wages and also collusion constraints are not implied by other constraints, they are both binding. However, it is not the case for upward collusion constraints. We have $w_\emptyset$ on the LHSs of these constraints meaning that either ex-post upward collusion constraint or ex-post downward collusion constraint should bind. When the input is monitored, ex-ante upward constraint is binding since there is no uncertainty in the output, the principal has to pay $w_\emptyset$ more ex-ante to prevent upward collusion. For the output monitoring, it depends on the monitoring cost. Because of the randomness of the output in the output monitoring, for ex-ante collusion the principal considers the combination of w_L and w_H wages with additional monitoring cost term. For ex-post upward collusion-proofness, we need to have a bigger w_H than $w_\emptyset$. If monitoring cost is lower than a limit, then ex-post upward collusion constraint is binding. We reach to the conclusion that ex-ante upward collusion constraint is not necessarily binding.

Next, we compare the efficiencies of both types of monitoring in terms of expected cost that the principal will face. For the sake of analysis, we focus on ex-post collusion-proof contract (Proposition 1-3) and both ex-post and ex-ante collusion-proof ones (Proposition 7-9).

Corollary 4 *If the same monitoring technology is available for both input and output monitoring (producing hard evidence of the variable that is monitored with the same probability μ) the principal prefers input monitoring, under which expected costs are lower. This continues to hold when ex-ante collusion possibilities can be ignored.*

At first sight, intuition might suggest that the principal should prefer output monitoring. Since the agent's effort does not produce high output efficiently due to to parameter π, there is a probability that the principal pays low output wages when the supervisor monitors output. However, agent's IC but supervisor's IC, both ex-ante collusion constraints and most importantly principal's expected cost function are modified when the input is monitored. The principal does not consider low output wages in the expected cost calculation for input monitoring since it is known that the report cannot be low output when the contracts are designed full collusion-proof. We infer that wages required for full collusion-proofness for the output monitoring are higher than the one for input monitoring.

The distinctive characteristic of the input monitoring analysis is the absence of parameter π. When this inefficiency variable is taken out of the model, all the related constraints are relaxed. For example; agent's IC becomes $w_H - w_L \geq \frac{c_e}{\mu}$ from $w_H - w_L \geq \frac{c_e}{\mu\pi}$. Since $\pi \in (0,1]$, when the supervisor monitor the agent's effort level, the supervisor can shrink the difference between $w_H - w_L$ to impose agent to put effort. It means that, now the principal can offer less to motivate the agent to induce effort.

In other words, the principal pays a higher wage in the"good" outcome under output monitoring (where the supervisor has evidence of high output) than in the "good" outcome under input monitoring (where the evidence of high effort is obtained) because the former outcome is less likely to happen than the latter: in the output monitoring case the agent gets the high wage if monitoring is successful and if high effort generates high output. For every time that the agent exerts high effort and monitoring is successful, he gets paid the high wage with probability π under output monitoring, with probability one under input monitoring. Thus, the wage for high output under output monitoring, w_H^*, must be higher than the wage for high effort under input monitoring. The principal might be unable to get back the rent he leaves to the agent because the agent is protected by limited liability (observe that w_L^X is zero and cannot be reduced any further) or by some collusion-proofness constraint. Although, we know that input monitoring is better option than output monitoring, the underlying reason behind this fact is not obvious and it is left to the corollary below.

Corollary 5 *Collusion constraints drive the extra cost that the principal incurs under output monitoring.*

Corollary 4 states the fact that output monitoring is costly under full-collusion proof contracts. An obvious question related to this result is whether limited liability constraints, collusion constraints or any other factor plays a role in this difference in expected costs. We solve the supervisor's minimization problem excluding all collusion constraints in appendix and show that expected costs are then come out equal. Therefore, any cost difference between input and output monitoring must relate to the collusion. It means that limited liability constraints do not generate cost difference between output and input monitoring. However, including collusion constraints make input monitoring cost efficient.

To find out the main reason behind the efficiency of input monitoring, we need to refer to the Corollary 3. We have shown that binding constraints may differ for input and output monitoring. For input monitoring, regardless of the effort and monitoring costs, ex-ante upward collusion constraint is always binding and ex-post upward collusion constraint is redundant since there is no uncertainty in the effort of the agent. However, we have two distinguishing cases for the output monitoring depending on the relationship between effort and monitoring cost. If monitoring cost is higher than a level, ex-ante upward collusion constraint suppresses ex-post one as happened in the input monitoring. In this case we end up with same expected cost for the principal. It can be seen from Table 1, full-collusion proof contracts, case (ii) for output monitoring compared with input monitoring. However, if we have $\frac{(1-\pi)}{\pi}c_e \geq c_m$, then ex-post upward collusion constraint becomes effective and output monitoring becomes more costly. Both upward collusion constraints puts a bound on $w_\emptyset$ but increase in this wage affect other wages through remaining constraints.

Main aim of the principal is to induce agent to exert effort. However, output monitoring considers the inefficiency of the agent's effort and if monitoring cost is very low, it takes the principal to adjust wages to prevent ex-post collusion. This is the underlying reason behind the cost difference between types of monitoring. The principal might be obliged to increase $w_\emptyset$ wage along with some other wages to prevent ex-post collusion which is not a concern in ex-ante collusion. Thus, these further raises in wages makes output monitoring inefficient.

We conclude that because the principal's objective is to induce high effort (input), providing incentive by directly monitoring the input should be less costly than providing effort incentives indirectly by monitoring output.

8 Concluding remarks

We analyse the optimal incentive schemes in multi-level hierarchical institutions by using a canonical agent-supervisor-principal model. We introduce supervision cost and extend this standard model. By usin this extented model, we introduce ex-ante collusion possibilities since acquiring information will have some cost to the supervisor. Even though ex-post collusions and their effects on the contracts have been widely investigated in the economics literature, ex-ante collusions are less known.

At first, after introducing ex-ante collusion constraints between two parties, we check whether there are some links between these constraints and ex-post collusion constraints. We show that preventing one type of collusion does not prevent the other, each of them should be treated separately. Other than these, we note that supervisor's incentive compatibility is automatically satisfied when the contracts are designed as full collusion-proof.

Furthermore, we show the effect of ex-ante collusion constraints on the optimal contracts and determine the channels leading to changes in both agent's and supervisor's wages. According to our results, there is a significant increase in expected costs and in some cases, they are even doubled.

Since expected costs are quite increased with newly introduced ex-ante constraints, we evaluate all four possible strategies the principal is able to use to reduce the expected costs: preventing both ex-post collusions, permitting downward ex-post collusion, permitting upward ex-post collusion and permitting both ex-post collusions. We compare expected costs for each resulting contracts and show that preventing all types of collusion is weakly dominant strategy for the principal. Vafai (2016) has shown that best strategy for the principal is to induce full ex-post collusion proof contracts when ex-ante collusions are not present in the model. As we extend the model, we further claim that best strategy would be to prevent all types of collusions in the hierarchy.

Lastly, we distinguish between two types of monitoring and conduct the input monitoring analysis along with the output monitoring. Our results prove that in this environment, input monitoring is efficient than output monitoring since ultimate purpose of the principal is to impose agent to put effort. For input monitoring we have one output, there is no uncertainty due to inefficiency of effort. Thus, ex-ante upward collusion constraint always dominates ex-post upward collusion constraint. However, this is not the case for output monitoring. Even if monitoring cost is so small, the principal has to put a higher limit to $w_\emptyset$ in output monitoring to prevent ex-post upward collusion. On the other hand, decrease in c_m proportionally reduces $w_\emptyset$ in input monitoring.

Ultimately, since the principal can ignore ex-post upward collusion due to certainty of the output there will be a surplus coming from $w_\emptyset$ difference compared to output monitoring. Therefore, delegated supervisor should monitor the effort level of the agent instead of the output that he produces when monitoring cost is low enough.

A possible future research can involve hierarchies with more than three levels or more than one agent. This could give us opportunity to check whether our claims with ex-ante collusion possibilities hold or not when peer monitoring or any other involvement happen. Also, as a robustness check similar analysis could be conducted with continuous effort ($e \in [0, 1]$) agent-supervisor-principal models. Moreover, as the data availability increases day by day, in the next decades, there might be sufficient number of incidents for us to test these models.

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Appendix

Proof of proposition 1: First, observe that $s_\emptyset$ appears in two constraints only, (3) and (4). It is easy to see that the principal must set $s_\emptyset^X = 0$, for if it is positive, $s_\emptyset$ can be reduced without violating these constraints, which leads to a fall in the expected cost in (1).

Second, observe also that w_H must be strictly positive and (2) must be binding. If (2) is not binding, the principal can reduce w_H by some ϵ, however small so that (2) continues to be satisfied without affecting any other constraint. Hence, $w_H^X = w_L^X + \frac{c_e}{\mu\pi}$. Through (5), this lower bound for w_H implies a lower bound for $w_\emptyset$, hence, $w_\emptyset^X = w_H^X$. So far we have established

$$w_H^X = w_\emptyset^X = w_L^X + \frac{c_e}{\mu\pi}.$$

Using this fact, (4) can be written as

$$s_L \geq \frac{c_e}{\mu\pi}. \tag{28}$$

It now follows that w_L must be set minimally, $w_L^X = 0$, because the only constraint where it appears, (2), will be softened by reducing w_L, to also reduce the expected cost for the principal. Thus, $w_\emptyset^X = \frac{c_e}{\mu\pi}$ and $w_H^X = \frac{c_e}{\mu\pi}$. The optimal agent contract is $(w_L^X, w_\emptyset^X, w_H^X) = (0, \frac{c_e}{\mu\pi}, \frac{c_e}{\mu\pi})$.

Now observe that the left hand side of (3), $\mu[\pi s_H + (1-\pi)s_L] \geq c_m$ appears also in the cost function (1). Clearly, cost minimization requires s_L and s_H be set at lowest possible values satisfying $\mu[\pi s_H + (1-\pi)s_L] \geq c_m$ and (28). There can be two cases according to whether the wages $s_L = \frac{c_e}{\mu\pi}$ and $s_H = 0$ satisfy $\mu[\pi s_H + (1-\pi)s_L] \geq c_m$ or not. So:

1. If $\frac{(1-\pi)}{\pi}c_e \geq c_m$, the optimal wages are $s_L^X = \frac{c_e}{\mu\pi}$, $s_H^X = 0$.
2. If $\frac{(1-\pi)}{\pi}c_e < c_m$, setting the wages as $s_L = \frac{c_e}{\mu\pi}$, $s_H = 0$ violates the supervisor's incentive compatibility constraint. Either s_L or s_H must be raised to satisfy that constraint with equality: $\mu[\pi s_H + (1-\pi)s_L] = c_m$. While the exact combination of the wage modification is not relevant for the principal's cost, the bounds in which the two wages could be adjusted can be determined, as follows: If $s_H = 0$, then s_L must be raised (above $\frac{c_e}{\mu\pi}$) to $\frac{c_m}{\mu(1-\pi)}$. Therefore, the range of adjustment for s_L^X is the interval $[\frac{c_e}{\mu\pi}, \frac{c_m}{\mu(1-\pi)}]$. On the other hand, if s_L is set minimally, $s_L = \frac{c_e}{\mu\pi}$, then $s_H = \frac{c_m}{\mu\pi} - \frac{(1-\pi)}{\pi}\frac{c_e}{\mu\pi}$. Therefore, the range of adjustment for s_H^X is the interval $[0, \frac{c_m}{\mu\pi} - \frac{(1-\pi)}{\pi}\frac{c_e}{\mu\pi}]$.

The supervisor's contract is thus as stated in the proposition. Plugging in the optimal wages in (1) yields the expected cost for the principal. **Q.E.D.**

Proof of proposition 2.a: Since $s_\emptyset$ is at the right hand side of all the constraints in which it appears, it is optimal to set $s_\emptyset^D = 0$.

Moving to the agent's wages, inspecting (4), (5) and (6) reveals that (4) must be binding, which means $w_\emptyset^D = w_H^D$, given $s_\emptyset^D = 0$. Using these findings, (6) becomes

$$\mu[\pi s_H + (1-\pi)s_L] + \mu(1-\pi)w_L \geq c_m + c_e + \mu(1-\pi)w_H.$$

To see that the agent's incentive-compatibility constraint (2) must be binding, observe that otherwise it would be possible to reduce w_H and economize on expected costs without violating any of the constraints. Thus, $w_H^D = w_L^D + \frac{c_e}{\mu\pi}$. Using this result, (6) now becomes

$$\mu[\pi s_H + (1-\pi)s_L] \geq c_m + \frac{c_e}{\pi} \tag{29}$$

This constraint implies the supervisor's incentive-compatibility constraint (3), which we shall drop after this point, leaving us with (2), (4), (5) and (29) as constraints, of which (2) and (4) are binding, with $s_\emptyset^D = 0$. Now, because the agent's wages do not appear in (29) and the remaining constraints are so far identical, the agent's optimal contract must be the same as the contract described in Proposition 1.

From this point on, the supervisor's optimal contract is derived using the same arguments as in Proposition 1, except that the constraint (3) is replaced by (29). The left hand side of (29) is an expected cost item in (1), which the principal would seek to minimize by binding, if possible, (29). It turns out that (29) must be binding. To see this, suppose on the contrary, that (29) is not binding. Then, to minimize costs the principal must be binding all other constraints by setting $s_L = \frac{c_e}{\mu\pi}$, $s_H = 0$. However, this violates (29), which becomes $0 \geq c_m + c_e$. We conclude that the optimal supervisor wages are non-negative numbers s_L^D and s_H^D satisfying $\mu[\pi s_H^D + (1-\pi)s_L^D] = c_m + \frac{c_e}{\pi}$, with minimum bounds given by $s_H = 0$ and $s_L = \frac{c_e}{\mu\pi}$ (from (4)). For $s_H = 0$ the principal can set $s_L = \frac{\pi c_m + c_e}{\mu\pi(1-\pi)}$, and for $s_L = \frac{c_e}{\mu\pi}$ the principal can set $s_H = \frac{c_m}{\mu\pi} - \frac{(1-\mu-\pi)}{\mu\pi}\frac{c_e}{\mu\pi}$. Substituting the optimal wages into (1) yields the minimum expected cost. **Q.E.D.**

Proof of proposition 2.b: Additional constraint (7) does not have $s_\emptyset$ variable in it. Therefore, we again set $s_\emptyset^U = 0$. Increasing w_H increases expected cost for principal and makes (7) and (5) harder to satisfy, hence we set $w_H^U = w_L^U + \frac{c_e}{\mu\pi}$ that is minimum wage it can be assigned due to agent's IC constraint. After that, we change w_H term in (7) with our finding. After a simplification, (7) becomes $w_\emptyset \geq \frac{c_m + c_e}{\mu} + w_L$. Also, we have another condition for $w_\emptyset$ that is $w_\emptyset \geq w_L + \frac{c_e}{\mu\pi}$ from (5). Now, to relax (4) and decrease expected for principal we should set $w_\emptyset^U$ to its minimum level. In this case, one of the above constraints should be binding, hence we have two cases to be analysed.

1. (5) is binding and (7) is not binding meaning that $\frac{c_e}{\mu\pi} \geq \frac{c_m+c_e}{\mu}$. When we simplify it, we have $\frac{(1-\pi)}{\pi}c_e \geq c_m$. If this is the case we should set $w_\emptyset^U = \frac{c_e}{\mu\pi} + w_L^U$. Then, we can readily set $w_L^U = 0$ resulting in $w_\emptyset^U = \frac{c_e}{\mu\pi}$ and $w_H = \frac{c_e}{\mu\pi}$. We directly set $s_L = \frac{c_e}{\mu\pi}$ and $s_H^U = 0$ and then check whether (3) is satisfied. When we put these into the equation, it gives us the same condition with choice of $w_\emptyset^U$ that is $\frac{(1-\pi)}{\pi}c_e \geq c_m$. It means that supervisor's IC is automatically satisfied. Therefore, indeed we should have $s_L^U = \frac{c_e}{\mu\pi}$ and $s_H^U = 0$.

2. (7) is binding and (5) is not binding meaning that $\frac{c_m+c_e}{\mu} \geq \frac{c_e}{\mu\pi}$. When we simplify it, we have $c_m \geq \frac{(1-\pi)}{\pi}c_e$. If this is the case, we should set $w_\emptyset^U = \frac{c_m+c_e}{\mu} + w_L^U$. By using this information, we get rid of w_L in LHS of (4). After that, there is no advantage of increasing w_L^U, so $w_L^U = 0$. We also acquire, $w_\emptyset^U = \frac{c_m+c_e}{\mu}$ and $w_H^U = \frac{c_e}{\mu\pi}$. If we ignore (3) for now, we set $s_L^U = \frac{c_m+c_e}{\mu}$. We should set $s_H^U = 0$ and then check whether (3) is satisfied. When we incorporate these s_L^U and s_H^U wages into the equation, it gives us the exact opposite condition with choice of $w_\emptyset^U$ that is $\frac{(1-\pi)}{\pi}c_e \geq c_m$. Therefore, these values do not satisfy supervisor's IC constraint automatically. Principal should increase either s_L^U or s_H^U to satisfy (3). If we still let $s_H^U = 0$, we can increase s_L^U from $\frac{c_m+c_e}{\mu}$ to $\frac{c_m}{\mu(1-\pi)}$. Also, we cannot decrease s_L^U below $\frac{c_m+c_e}{\mu}$. Therefore, $s_L^U = [\frac{c_m+c_e}{\mu}, \frac{c_m}{\mu(1-\pi)}]$. On the other hand, we could set s_L^U to the minimum level restricted by (4) and increase s_H. When $s_L^U = \frac{c_m+c_e}{\mu}$ we should set $s_H^U = \frac{c_m}{\mu} - \frac{(1-\pi)}{\mu\pi}$. Therefore, we have $s_H^U = [\frac{c_m}{\mu} - \frac{(1-\pi)}{\mu\pi}, 0]$. **Q.E.D.**

Proof of proposition 3: We have 7 constraints in total. Decreasing $s_\emptyset$ softens related constraints, so $s_\emptyset = 0$. w_H exist in LHS of (6) and w_L is in LHS of (4). Therefore, it is better to proceed with adjustment of $w_\emptyset$. Setting $w_\emptyset$ as small as possible is better since it softens (4) and (6). There are two constraint that can bind $w_\emptyset$, those are (5) and (7). Both cannot bind at the same time, hence there are two cases to be analysed.

1. (5) is binding and (7) is not binding meaning that $w_H \geq \frac{c_m}{\mu} + [\pi w_H + (1-\pi)w_L]$. Then, we should set $w_\emptyset = w_H$. With this information (6) becomes $\mu[\pi s_H + (1-\pi)s_L] + \mu(1-\pi)w_L \geq \mu(1-\pi)w_H + c_m + c_e$. After this, w_H goes to the RHS, hence $w_H = w_L + \frac{c_e}{\mu\pi}$. By changing w_H, (6) comes out as $\mu[\pi s_H + (1-\pi)s_L] \geq \frac{c_e}{\pi} + c_m$. Supervisor's IC constraint will not bind and it can be ignored after this point. Simplified version of constraint permits us to set $w_L = 0$ directly. Other agent's wages are $w_\emptyset = \frac{c_e}{\mu\pi}$ and $w_H = \frac{c_e}{\mu\pi}$. At the beginning, we made assumption about binding constraint, it should be checked with the wages we claimed. When we incorporate them into

$w_H \geq \frac{c_m}{\mu} + [\pi w_H + (1-\pi)w_L]$, it is simplified as $\frac{1-\pi}{\pi}c_e \geq c_m$. After that we check whether $s_L = \frac{c_e}{\mu\pi}$ and $s_H = 0$ directly satisfy (6). It comes out as $-c_e \geq c_m$ which is not possible. Therefore, principal has to increase s_L and s_H wages to satisfy (6) such that $\mu[\pi s_H + (1-\pi)s_L] \geq \frac{c_e}{\pi} + c_m$ while $s_L \geq \frac{c_e}{\mu\pi}$. At the end, $s_L = [\frac{c_e}{\mu\pi}, \frac{\pi c_m + c_e}{\mu\pi(1-\pi)}]$ and $s_H = [\frac{c_m}{\mu\pi} - \frac{(1-\mu-\pi)}{\mu\pi}\frac{c_e}{\mu\pi}, 0]$.

2. (7) is binding and (5) is not binding. In that case, we should have $\frac{c_m}{\mu} + [\pi w_H + (1-\pi)w_L] \geq w_H$. We set $w_\emptyset = \frac{c_m}{\mu} + [\pi w_H + (1-\pi)w_L]$. Incorporating this to (6) results in $[\pi s_H + (1-\pi)s_L] \geq 2c_m + c_e$, hence (3). After that, we set $w_H = w_L + \frac{c_e}{\mu\pi}$. putting this information into $w_\emptyset$ gives $w_\emptyset = \frac{c_m}{\mu} + \frac{c_e}{\mu} + w_L$. Setting $w_\emptyset$ in (4) cancels w_L which allows us to set $w_L = 0$. Also, $w_H = \frac{c_e}{\mu\pi}$ and $w_\emptyset = \frac{c_m}{\mu} + \frac{c_e}{\mu}$. When we check our assumption on $w_\emptyset$ at the beginning, it gives the condition $\frac{1-\pi}{\pi}c_e \leq c_m$. We continue with checking $s_L = \frac{c_m+c_e}{\mu}$ and $s_H = 0$ satisfy (6) or not. Putting them into the (6) simplifies as $\frac{1-\pi}{\pi}c_e \geq c_m$ that is the opposite condition that we are operating now. Therefore, these wages do not satisfy ex-ante downward constraint automatically and principal should increase any of them such that $\mu[\pi s_H + (1-\pi)s_L] = 2c_m + c_e$ while $s_L \geq \frac{c_m+c_e}{\mu}$. At the end, $s_L = [\frac{c_m+c_e}{\mu}, \frac{2c_m+c_e}{\mu(1-\pi)}]$ and $s_H = [\frac{(1+\pi)c_m+\pi c_e}{\mu\pi}, 0]$. **Q.E.D.**

Proof of proposition 4: Set $w_L = 0$, bribing constraint becomes redundant since $w_\emptyset \geq w_H \geq k$. Also, we can choose $s_L = 0$ and ex-post downward collusion constraint becomes irrelevant as well. With this information, $s_\emptyset = 0$ becomes obvious. Remaining constraints are as follows

$$w_\emptyset \geq w_H.$$

$$w_H \geq \frac{c_e}{\mu\pi}.$$

$$\mu\pi s_H + w_\emptyset(\mu - \mu\pi) \geq c_m.$$

$$\mu\pi s_H + \mu\pi w_H \geq c_m + c_e + \mu\pi w_\emptyset.$$

$$w_\emptyset \geq \frac{c_m}{\mu} + \pi w_H.$$

If we rewrite supervisor's IC constraint as $\mu\pi s_H + \mu\pi w_\emptyset \geq c_m + \mu\pi w_\emptyset$ and since $w_\emptyset \geq w_H$, we can say that ex-ante downward collusion constraint is harder to satisfy. Thus, supervisor's IC becomes irrelevant afterwards. We need to choose minimal wage for $w_\emptyset$ since it lowers the expected cost. We have two constraints that can determine lower bound for $w_\emptyset$: either ex-post upward collusion constraint or ex-ante upward collusion constraint.

1. Assume that ex-post upward collusion constraint is binding. Then, we should have $w_\emptyset \geq w_H \geq \frac{c_m}{\mu} + \pi w_H$ and $w_\emptyset = w_H$. Increasing w_H increases the expected cost,

so $w_H = \frac{c_e}{\mu\pi}$. Incorporate this to the preceding inequality, we are in case $\frac{(1-\pi)}{\pi}c_e \geq c_m$. Other wages are $w_\emptyset = \frac{c_e}{\mu\pi}$ and $s_H = \frac{c_m+c_e}{\mu\pi}$.

2. If ex-ante upward collusion constraint is binding, we have $w_\emptyset \geq \pi w_H + \frac{c_m}{\mu} \geq w_H$ and $w_\emptyset = \pi w_H + \frac{c_m}{\mu}$. Ex-ante downward collusion constraint becomes $s_H + (1-\pi)w_H \geq c_m \frac{1+\pi}{\mu\pi} + \frac{c_e}{\mu\pi}$. We need to set $w_H = \frac{c_e}{\mu\pi}$ since one unit increase in s_H and w_H costs same but s_H helps more to satisfy ex-ante downward collusion constraint. Thus, we can prevent ex-ante downward collusion by offering less wage. We end up in case $\frac{(1-\pi)}{\pi}c_e \leq c_m$. Other constraints are $w_\emptyset = \frac{c_m+c_e}{\mu}$ and $s_H = c_m \frac{1+\pi}{\mu\pi} + \frac{c_e}{\mu}$. **Q.E.D.**

Proof of proposition 5: Begin bey setting $s_\emptyset = 0$. Ex-post upward collusion constraint implies bribing constraint. Simplified remaining constraints are

$$w_H \geq w_\emptyset + k.$$

$$w_\emptyset \geq w_L + \frac{c_e}{\mu\pi}.$$

$$\mu\pi s_H + s_L(\mu - \mu\pi) + \mu\pi w_H \geq c_m + \mu\pi w_\emptyset.$$

$$s_L + w_L \geq w_\emptyset.$$

$$\mu\pi s_H + \mu\pi w_H + s_L(\mu - \mu\pi) + w_L(\mu - \mu\pi) \geq c_m + c_e + \mu w_\emptyset.$$

$$w_\emptyset \geq \frac{c_m}{\mu - \mu\pi} + w_L.$$

Since we need to choose minimal $w_\emptyset$, we will have two cases again. Either agent's IC or ex-ante upward collusion constraint is binding.

1. Assume that binding constraint is agent's IC, we are in case $\frac{(1-\pi)}{\pi}c_e \geq c_m$. Set $w_\emptyset = w_L + \frac{c_e}{\mu\pi}$. Now, all the w_L wages are on the RHSs of the remaining constraints, so $w_L = 0$, $w_\emptyset = \frac{c_e}{\mu\pi}$. Downward ex-ante collusion constraint implies supervisor's IC. At the end we need to set $\mu\pi w_H + (\mu - \mu\pi)s_L + \mu\pi s_H = c_m + c_e + \frac{c_e}{\pi}$ while two of these wages have a lower bound.

2. If upward ex-ante upward collusion constraint is binding. Set $w_\emptyset = w_L + \frac{c_m}{\mu - \mu\pi}$ and we have $\frac{(1-\pi)}{\pi}c_e \leq c_m$, exact opposite case. Again setting $w_L = 0$ is optimal and ex-ante downward collusion constraint implies supervisor's IC constraint. For cost minimization, we set $\mu\pi w_H + (\mu - \mu\pi)s_L + \mu\pi s_H = c_m + c_e + \frac{c_m}{\mu - \mu\pi}$. Just as in the case above, we have some lower bounds on two wages as well. **Q.E.D.**

Proof of proposition 6: Setting w_L and s_L to their lowest bound softens all the related constraints, so that we begin with $s_L = 0$, $w_L = 0$. Since we have $s_\emptyset + w_\emptyset \geq$

$w_\emptyset \geq \frac{c_e}{\mu\pi} \geq k$, (8) and (9) are redundant. For the remaining constraints, $s_\emptyset$ wage is always in RHSs, set $s_\emptyset = 0$. (14) becomes redundant since (15) is harder to satisfy. Simplified version of he remaining constraints are as follows:

$$w_H \geq w_\emptyset + k.$$

$$w_\emptyset \geq \frac{c_e}{\mu\pi}.$$

$$\mu\pi s_H + \mu\pi w_H \geq c_m + w_\emptyset(2\mu\pi - \mu).$$

$$\mu\pi s_H + \mu\pi w_H \geq c_m + c_e + \mu\pi w_\emptyset.$$

$$w_\emptyset \geq \frac{c_m}{(\mu - \mu\pi)}.$$

Supervisor's IC constraint (21) becomes redundant since ex-ante downward collusion constraint (22) puts a higher bound on combination of s_L and s_H. Either (20) or (23) should determine the bound for $w_\emptyset$; we have two cases to be analysed.

1. Assume that (20) is binding, $w_\emptyset \geq \frac{c_e}{\mu\pi} \geq \frac{c_m}{(\mu-\mu\pi)}$. We are in case $\frac{1-\pi}{\pi}c_e \geq c_m$. We set $w_\emptyset = \frac{c_e}{\mu\pi}$ to minimize the expected cost of the principal. Also, for optimization we need $s_H + w_H = \frac{c_m+2c_e}{\mu\pi}$ while $w_H \geq \frac{c_e}{\mu\pi} + k$.

2. When (23 is binding, we need to have $\frac{1-\pi}{\pi}c_e \leq c_m$. Setting $w_\emptyset = \frac{c_m}{(\mu-\mu\pi)}$ is optimal. To minimize the expected cost, the principal sets $s_H + w_H = \frac{c_m}{\mu\pi(1-\pi)} + \frac{c_e}{\mu\pi}$ while $w_H \geq \frac{c_m}{\mu-\mu\pi} + k$. **Q.E.D.**

Proof of corollary 1: To prove this corollary, we need to compare the expected costs for the principal between proposition 3 and proposition 4,5 and 6 separately. Condition separating two cases is same in each proposition, so we compare the costs for each case separately.

Comparison of case *(i)*: All the expected costs are same, $EC = c_m + c_e\frac{(1+\mu\pi)}{\mu\pi}$, if $\frac{(1-\pi)}{\pi}c_e \geq c_m$ that is monitoring cost is below a level.

Comparison of case *(ii)*: If $\frac{(1-\pi)}{\pi}c_e \leq c_m$, expected cost of full collusion proof contracts and downward collusion permitted contracts are same. Also, expected cost of upward collusion permitted and both collusions permitted contracts are same. Assume that full collusion proof is cost efficient, $c_m + c_e + \frac{c_m}{\mu-\mu\pi} \geq c_m\frac{(1+\mu)}{\mu} + c_e\frac{1+\mu}{\mu}$. After a simplification, we acquire $\frac{(1-\pi)}{\pi}c_e \leq c_m$ meaning that full collusion proof contracts cost less. **Q.E.D.**

Proof of proposition 7: Set $s_\emptyset^X = 0$ since all $s_\emptyset$ variables are in RHSs of the constraints. (25) becomes $s_H \geq \frac{c_m}{\mu}$ and (4) becomes $s_L + w_L \geq w_\emptyset$. Set $w_\emptyset^X = w_H^X$ and $w_H^X = w_L^X + \frac{c_e}{\mu}$ since reducing $w_\emptyset$ and w_H decreases expected cost for principal and also relaxes (4) so that we set them to their lowest bounds. We have $w_H^X = w_\emptyset^X = w_L^X + \frac{c_e}{\mu}$. Incorporating this to the (4), we acquire $s_L \geq \frac{c_e}{\mu}$. Then, there is no bound on w_L, so we set $w_L^X = 0$, $w_\emptyset^X = \frac{c_e}{\mu}$ and $w_H^X = \frac{c_e}{\mu}$. Remaining variables come out as $s_L^X = \frac{c_e}{\mu}$, $s_\emptyset^X = 0$ and $s_H^X = \frac{c_m}{\mu}$. **Q.E.D.**

Proof of proposition 8.a: Reducing $s_\emptyset$ relaxes (25), (4) and (26), set $s_\emptyset^D = 0$. We then adjust $w_\emptyset^D = w_H^D$ since decreasing $w_\emptyset$ relaxes (4) and (26). Also, it reduces EC. With these findings, (26) becomes $s_H \geq \frac{c_m+c_e}{\mu}$.

Resulting that new ex-ante downward collusion constraint suppresses new supervisor incentive compatibility constraint which makes it redundant. We have $w_H \geq w_L + \frac{c_e}{\mu}$, $s_L + w_L \geq w_H$, $s_H \geq \frac{c_m+c_e}{\mu}$. Increasing w_L leads to an increase in w_H, thus it is not optimal. Setting $w_L^D = 0$ gives $w_H^D = \frac{c_e}{\mu}$ and $s_L^D = \frac{c_e}{\mu}$. At last, we set $s_H^D = \frac{c_m+c_e}{\mu}$ that is its lowest bound and $w_\emptyset^D = \frac{c_e}{\mu}$. **Q.E.D.**

Proof of proposition 8.b: (27) puts a bigger bound on $w_\emptyset$ than (5). Therefore, (5) is implied by (27). Set $w_\emptyset^U = w_H^U + \frac{c_m}{\mu}$, $w_H^U = w_L^U + \frac{c_e}{\mu}$ to their minimum bounds. We directly set $s_\emptyset^U = 0$ and using this as information $s_H = \frac{c_m}{\mu}$. Increasing w_L does not change the $w_H - w_L$, so that it is optimal to set to minimum value restricted by limited liabilities. $w_L^U = 0$, $w_H = \frac{c_e}{\mu}$, $w_\emptyset = \frac{c_m+c_e}{\mu}$. Then, $s_L = \frac{c_m+c_e}{\mu}$ when we set it minimally. **Q.E.D.**

Proof of proposition 9: (5) is redundant. Set $w_\emptyset = w_H + \frac{c_m}{\mu}$ and $s_\emptyset = 0$. (26) becomes $s_H \geq \frac{c_e+2c_m}{\mu}$ and (25) becomes $s_H \geq \frac{c_m}{\mu}$. Then, (25) is redundant. Remaining constraints are $w_H - w_L \geq \frac{c_e}{\mu}$, $s_H \geq \frac{c_e+2c_m}{\mu}$, $s_L + w_L \geq w_H + \frac{c_m}{\mu}$. We set w_H minimally to $w_H = w_L + \frac{c_e}{\mu}$. To minimize expected cost, we have $w_L = 0$. At the end, setting simplified constraints to their minimum gives $s_H = \frac{c_e+2c_m}{\mu}$, $w_H = \frac{c_e}{\mu}$, $w_\emptyset = \frac{c_e+c_m}{\mu}$. **Q.E.D.**

Proof of corollary 2: Looking at the proposition 3 and 6, we have 2 cases for the output monitoring and single case for the input monitoring.

Output monitoring case *(i)*: (5) dominates (7), so $w_\emptyset \geq w_H$. From (2), we have $w_H \geq w_L + \frac{c_e}{\mu\pi}$. Combining these information, we end with the relationship $w_\emptyset \geq w_H \geq w_L + \frac{c_e}{\mu\pi}$. We can say that any combination of s_H and s_L will be smaller than $w_\emptyset$, so $w_\emptyset \geq piw_H + (1-\pi)w_L$. Using this information in (6) yields $\mu[\pi s_H + (1-\pi)s_L] \geq \mu s_\emptyset + c_m + c_e \geq s_\emptyset + c_m$.

Output monitoring case *(ii)*: Inserting (7) into (6) directly gives us $\mu[\pi s_H + (1-\pi)s_L] \geq \mu s_\emptyset + 2c_m + c_e \geq s_\emptyset + c_m$.

Input monitoring: Using inequality (27) in (26), results in $s_H - s_\emptyset \geq \frac{2c_m+c_e}{\mu} \geq \frac{c_m}{\mu}$. **Q.E.D.**

Proof of corollary 3: Ex-post and ex-ante downward collusion constraints are independent of each other. Regardless of type of monitoring they are binding. When we look at upward collusion constraints, we distinguish between monitoring types.

1) When the supervisor monitors the input ex-post upward collusion constraint is suppressed by ex-ante upward collusion constraint.

2) In the case of output monitoring, depending on the effort and monitoring cost either ex-post upward or ex-ante upward binds. Naturally, if monitoring cost is too much, binding constraint will be ex-ante one since it requires to calculate the supervisor's expected utility surpluss coming from not working. **Q.E.D.**

Proof of corollary 4: We begin with the comparison of expected costs for ex-post collusion-proof contracts. Then we look at full-collusion proof contracts, so that we have two cases to be analysed.

1. Assume that output monitoring is more costly compared to input monitoring in the case of ex-post collusion-proof contracts. Then we should have $\frac{c_e}{\mu\pi} \geq c_m + \frac{c_e}{\mu}$ if $\frac{1-\pi}{\pi}c_e \geq c_m$. Interchanging the terms and taking out them to gives $\frac{1-\pi}{\mu\pi}c_e \geq c_m$. And since $\mu \in (0,1]$, $\frac{1-\pi}{\mu\pi}c_e \geq \frac{1-\pi}{\pi}c_e$ which satisfies our if condition.

Also, we need $c_m + \frac{1-\mu+\mu\pi}{\mu\pi}c_e \geq c_m + \frac{c_e}{\mu}$ if $\frac{1-\pi}{\pi}c_e \leq c_m$. Simplifying the inequality gives us $\frac{1}{\pi} - \frac{\mu}{\pi} + \mu \geq 1$ and then we have $1 \geq \pi$. Since this is true, our assumption holds. For ex-post collusion-proof contracts, output monitoring is more costly to conduct.

2. Assume that output monitoring is more costly compared to input monitoring in the case of full collusion-proof contracts. Then we should have $c_m + \frac{1+\mu\pi}{\mu\pi}c_e \geq \frac{1+\mu}{\mu}c_m + \frac{1+\mu}{\mu}c_e$. Simplification brings us to the condition that we are in: $\frac{1-\pi}{\pi}c_e \geq c_m$ meaning that our assumption holds.

If $\frac{1-\pi}{\pi}c_e \leq c_m$ both input monitoring and output monitoring have same expected cost. Therefore, input monitoring weakly dominates output monitoring. **Q.E.D.**

Proof of corollary 5: To prove this statement, we derive contracts without collusion constraints. If the expected costs come out equal, then we conclude that extra costs are due to collusion constraint, otherwise limited liabilities would be effective.

First, we solve the minimization problem of the principal under output monitoring subject to (1), (2) and (3). Optimal contracts satisfying limited liabilities with incentive compatibility constraint becomes $(w_L, w_\emptyset, w_H) = (0, 0, \frac{c_e}{\mu\pi})$, $(s_L, s_\emptyset, s_H) = (\sigma_L, 0, \sigma_H)$ such that $\mu[\pi\sigma_H + (1-\pi)\sigma_L] = c_m$ and $EC = c_e + c_m$.

Solving for the same problem under input monitoring results in: $(w_L, w_\emptyset, w_H) = (0, 0, \frac{c_e}{\mu})$, $(s_L, s_\emptyset, s_H) = (0, 0, \frac{c_m}{\mu})$ and $EC = c_e + c_m$.

Since expected costs are same when the collusion-proofness constraints are excluded and as we proved in the preceding corollary, output monitoring is costly than input monitoring when the principal prevents collusions, we conclude that the difference in expected costs are because of collusion constraints. **Q.E.D.**